A survey on random Laplacian eigenfunctions
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At the end of the 18th century, Ernst Chladni, a physicist and musician, made an interesting discovery: he observed that when he excited a metal plate with the bow of his violin, he could hear sounds of different frequency. The plate was fixed only at its center, and when Chladni put some sand on it, then for each frequency a curious pattern appeared, today known as Chladni figures. Some time later, Kirchhoff pointed out that these patterns correspond to nodal sets of eigenfunctions of the biharmonic operator.

In this talk, we lead a survey over the random Laplacian eigenfunctions on a manifold $\mathcal{M}$, that are, functions f defined on $\mathcal{M}$ that satisfy the Helmholtz equation, $\Delta f + Ef = 0$, for some fixed eigenvalue $E > 0$, where Δ denotes the Laplace operator. For such functions, we then consider their zero set. Typical examples that are well-studied arise when considering $\mathcal{M}$ to be the plane, the sphere, or the torus. We point out local quantities and global quantities that are worth to be analysed, such as number of connected components, or the volumes of the zero sets and are interested in limit theorems for these quantities as the eigenvalue goes to infinity. If time permits, we will also give some elements of the key techniques that allow one to obtain such limit theorems.