Sobolev mappings: from liquid crystals to irrigation via degree theory.

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Sobolev spaces are a natural framework for the analysis of problems in partial differential equations and calculus of variations. Some physical and geometric contexts, such as liquid crystals models and harmonic maps, lead to consider Sobolev maps, that is, Sobolev vector functions whose range is constrained in a surface or submanifold of the space. This additional nonlinear constraint provokes the appearance of finite-energy topological singularities. These singularities are characterized by a nontrivial topological invariant such as the topological degree. They can be imagined to change the topology of the domain of the map by drilling holes. They can represent an obstruction to the strong approximation by smooth maps. Finally they represent source and sink terms in an optimal transportation or irrigation problem arising in the study of the weak approximation and of the relaxed energy.