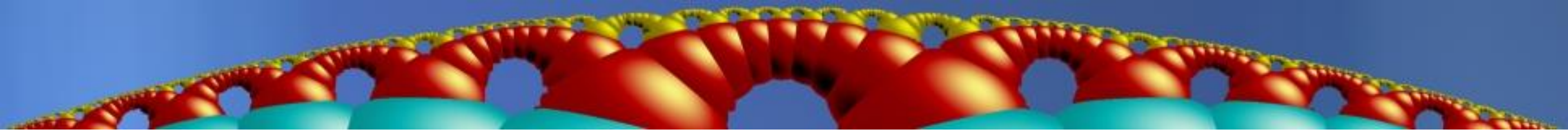


Mathematical Imagery

Jos Leys

BSSM
Brussels, August 2015

Who am I ?

- Retired engineer
- Hobby: math visualization
- www.josleys.com

Mathematical Imagery by Jos Leys

Home | Galleries | Articles | References | Forum | Links | About

Latest Galleries :

- 3D Newton fractals
- Mandelbrot tribute 2 ...through another 57 images in color.
- Tribute to Benoît Mandelbrot ...through 51 images.
- Kaleidoscopic IFS 32 amazing fractal 3D objects.
- Mandelbox A special kind of 3D fractal.

View all galleries

Stay informed on updates to this site using RSS

Newton type fractals in 3D

Posted on 2011-01-17 18:17

Newton's method for finding the roots of an equation creates fractals if the variable in the equation is a complex number. I found a way to show these fractals in 3D. See this new gallery.

Another tribute to Benoît Mandelbrot

Posted on 2010-11-09 12:55

I further improved my algorithm for 3d views of Mandelbrot and Julia sets. See this gallery.

A tribute to Benoît Mandelbrot.

Posted on 2010-10-24 16:52

Benoît Mandelbrot passed away on October 14, 2010.
He will be remembered in the first place for his contributions to science, but he also gave us the magnificent beauty of the Mandelbrot set. As a tribute, I have compiled a collection of images of the set in a 3D form. See this gallery.

Kaleidoscopic 3D IFS fractals

Posted on 2010-09-07 23:52

As a follow-up to the article, ("A fractal soccer ball"), about "kaleidoscopic 3D Iterated Function System fractals", I have now opened a new gallery with 32 images.

A fractal soccer ball

Posted on 2010-06-21 12:45

A new article, "A fractal soccer ball", about "kaleidoscopic 3D IFS fractals", was published on the site of the CNRS.

Dimensions wins the Prix d'Alembert.

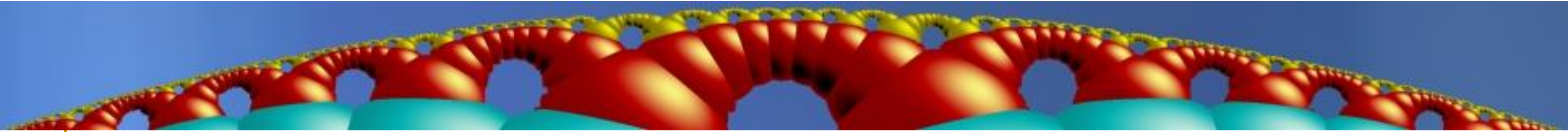
Posted on 2010-06-21 12:30

The film "Dimensions", that I made together with Etienne Ghys and Aurélien Alvarez, won the "Prix d'Alembert a bi-annual price given by the French Mathematical Society to the best projects in math vulgarization.

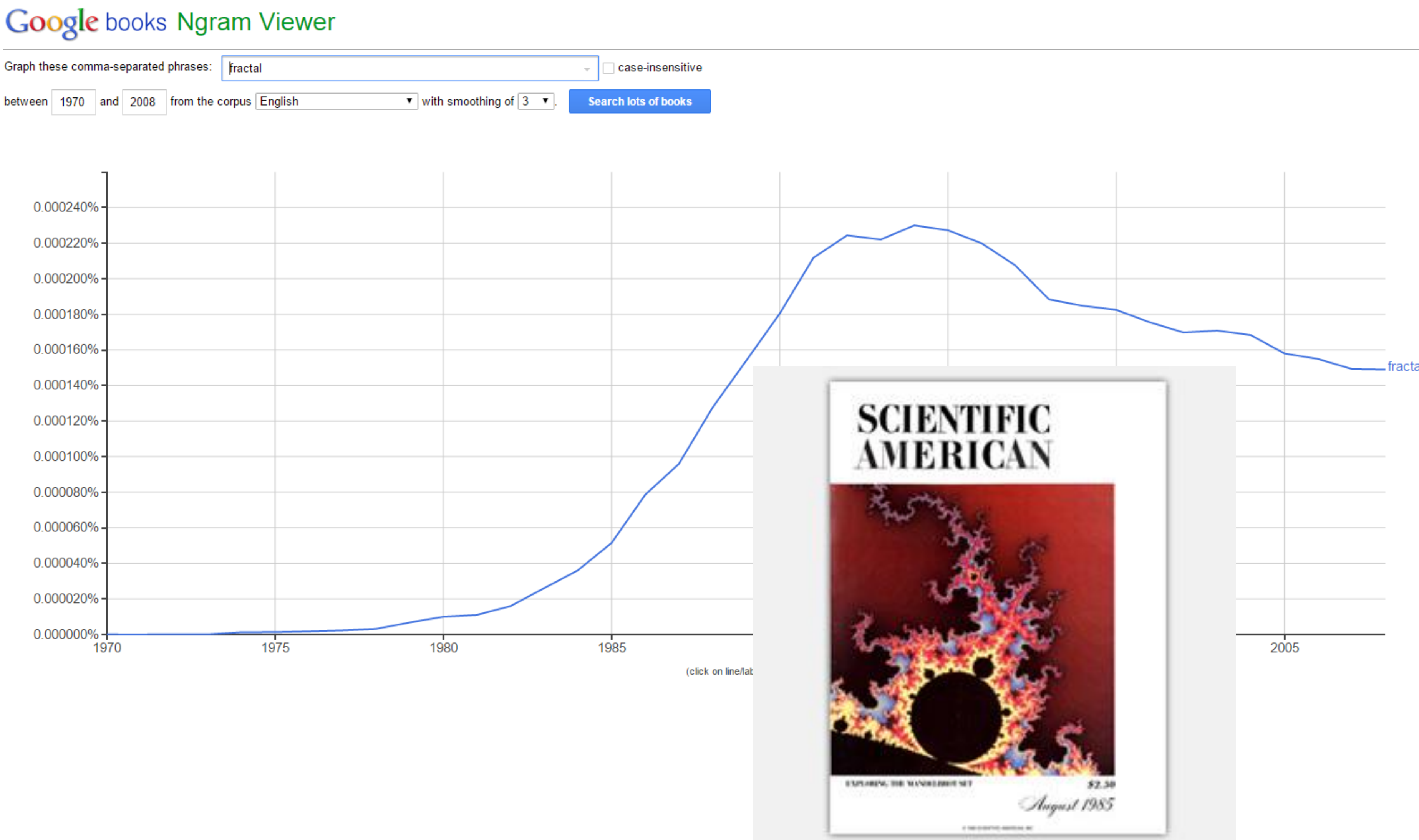
The Mandelbox

Posted on 2010-05-28 11:53

I wrote another article (again only in French, sorry!) about this special kind of 3D fractal.
There now is a small new gallery with some images on this subject..



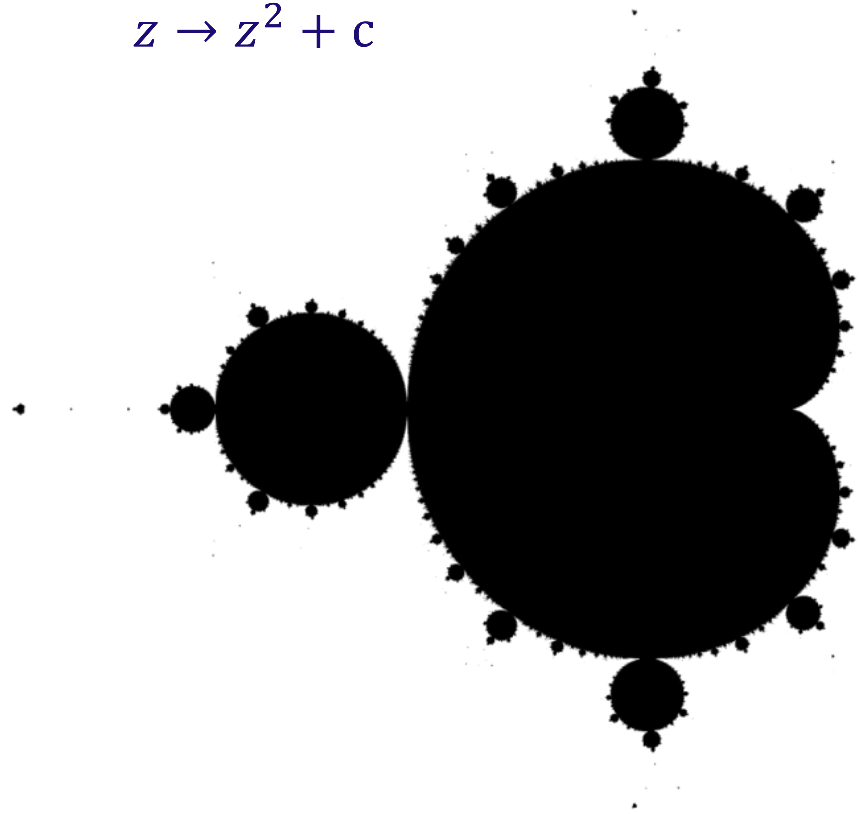
Occurence of the word ‘fractal’ in books 1970-2008

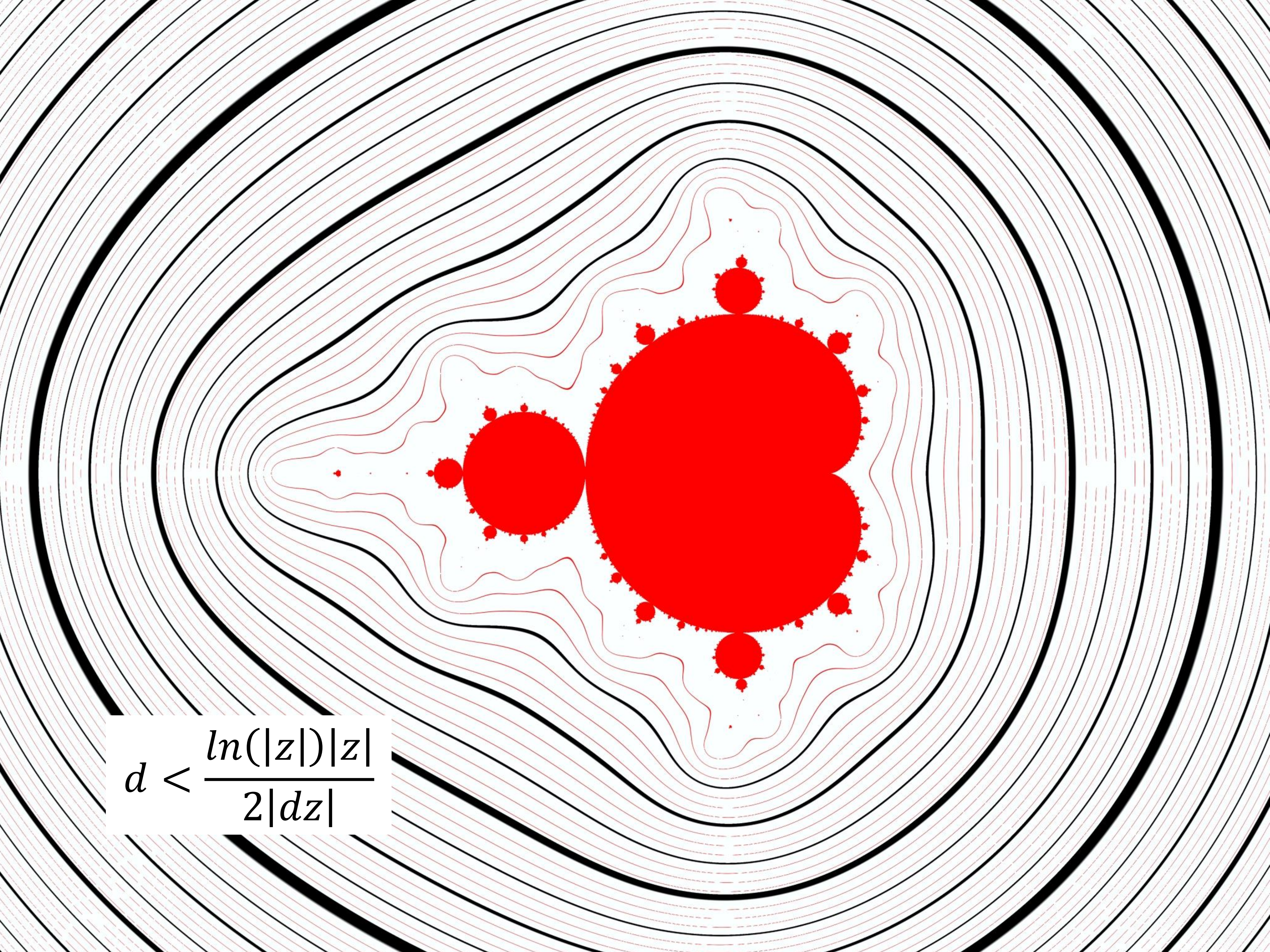


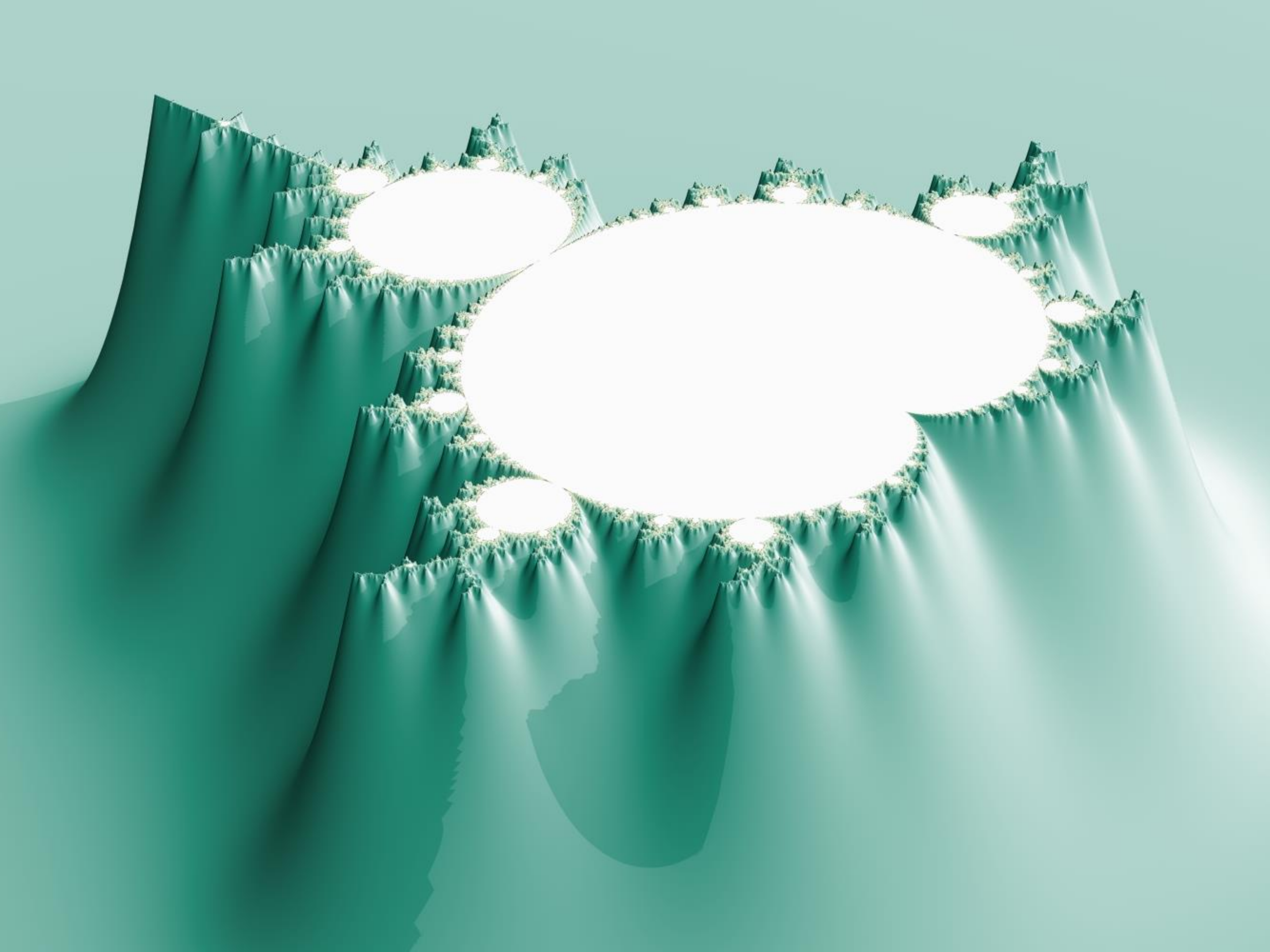
“The holy grail”

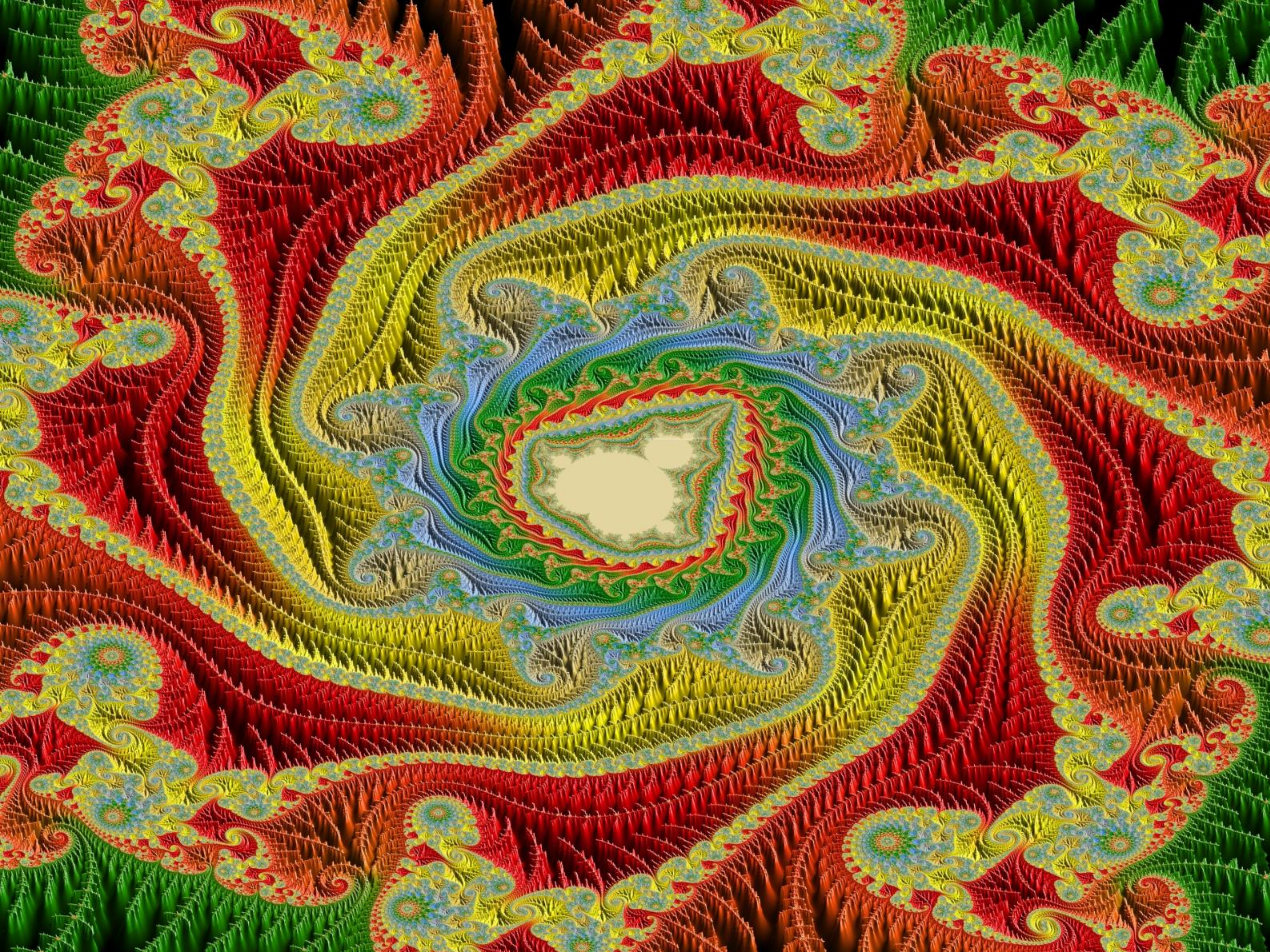
- Finding a 3D equivalent of the Mandelbrot set

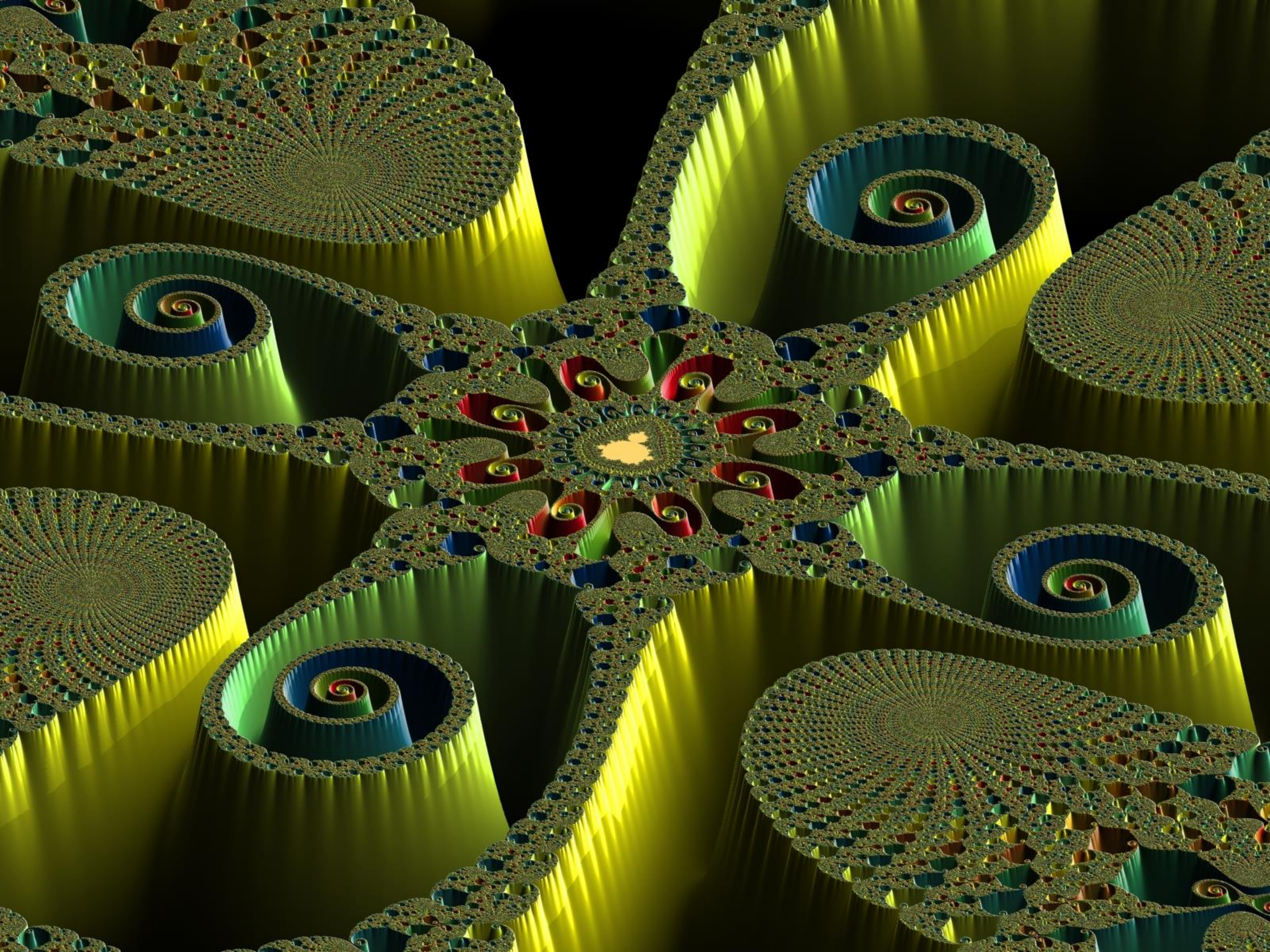
$$z \rightarrow z^2 + \mathrm{c}$$



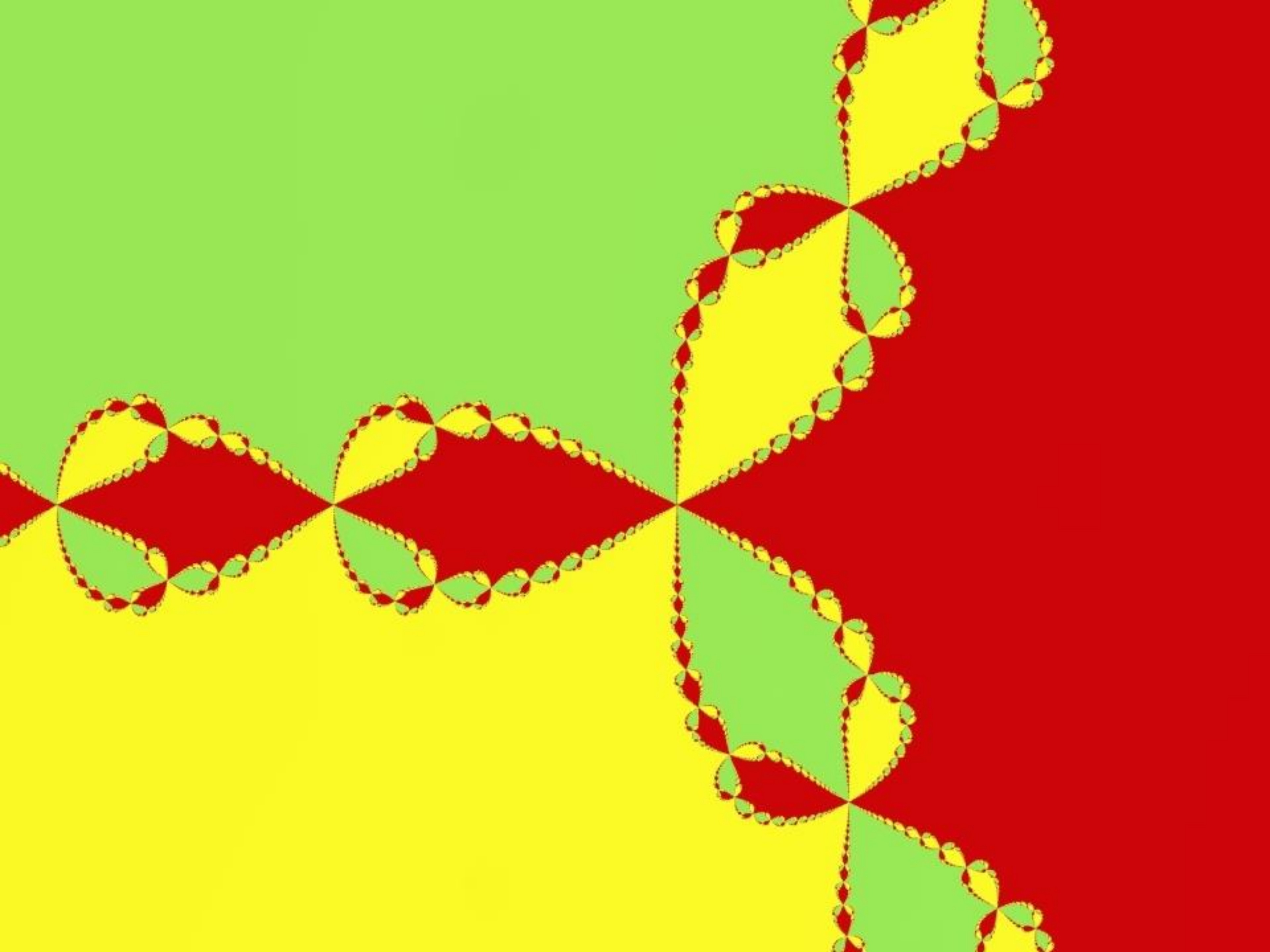

$$d < \frac{ln(|z|)|z|}{2|dz|}$$

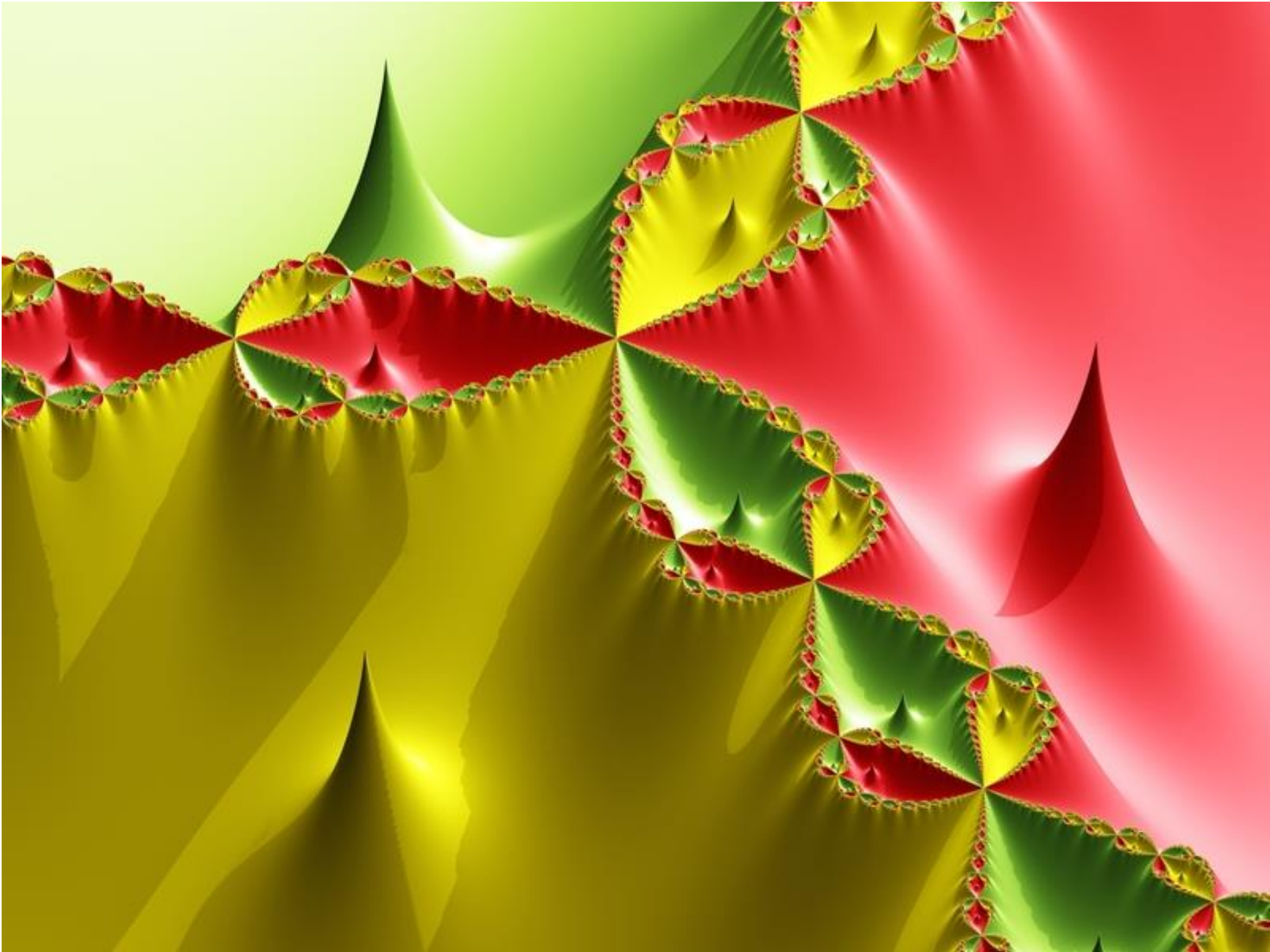


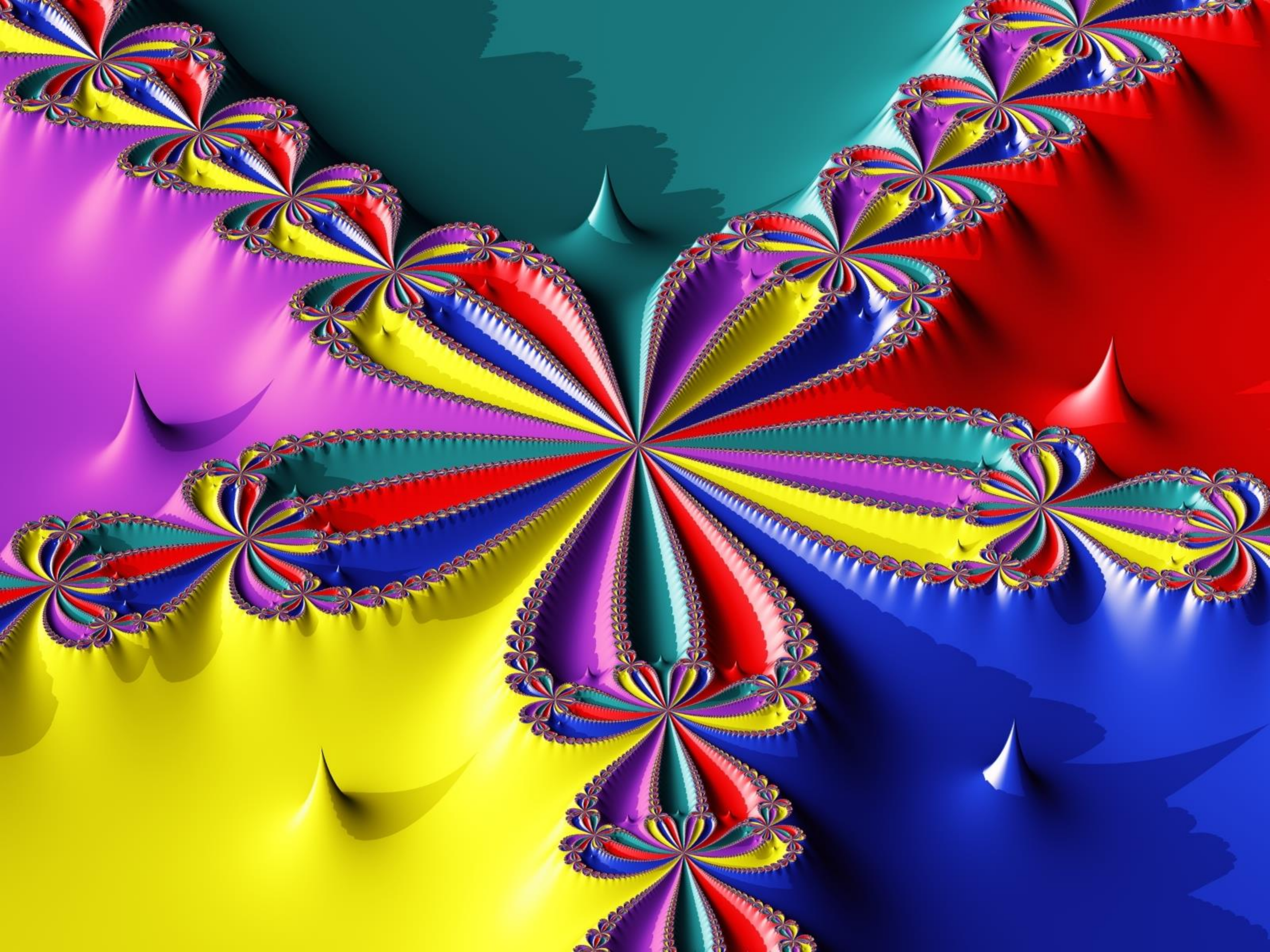


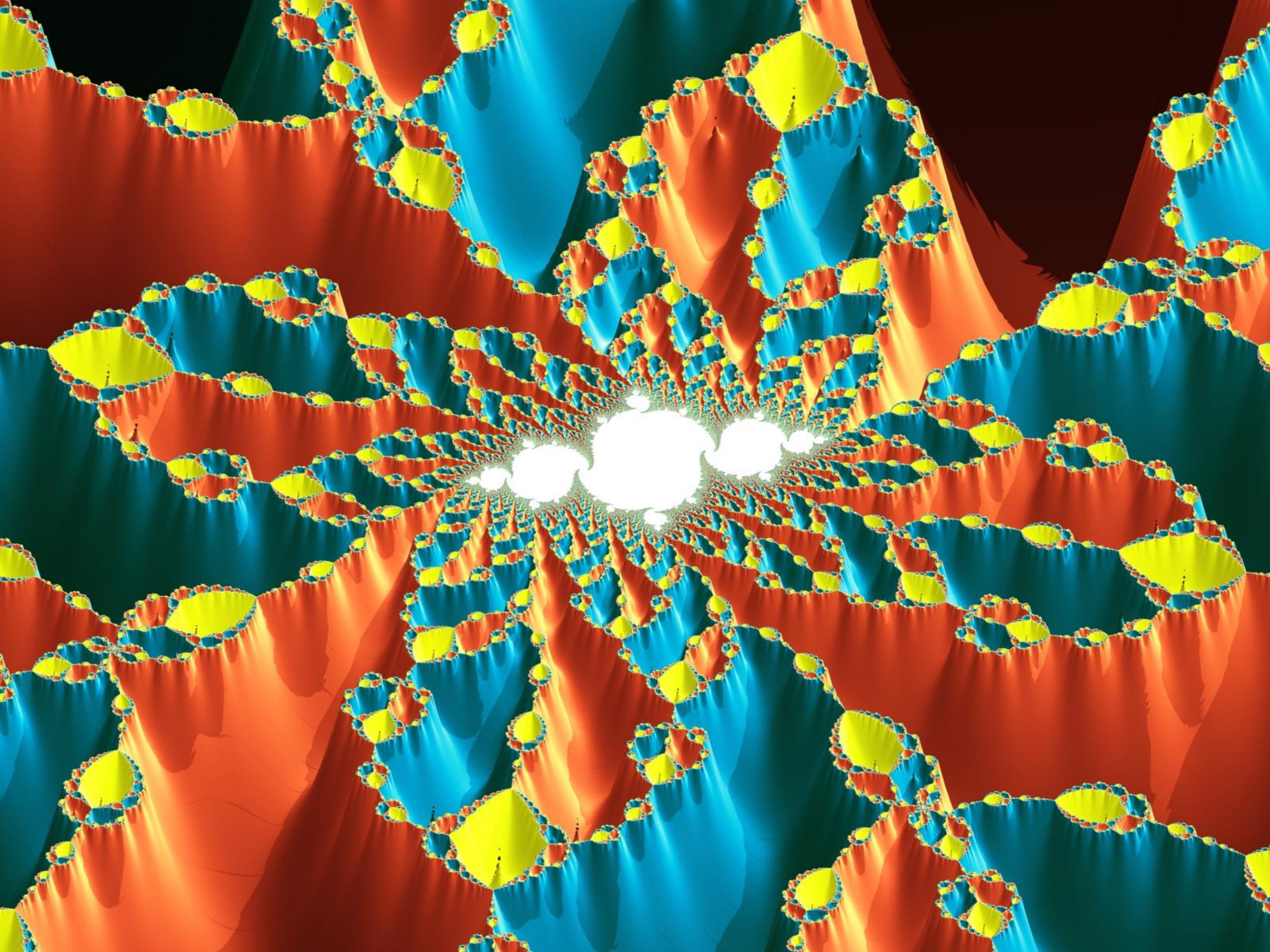


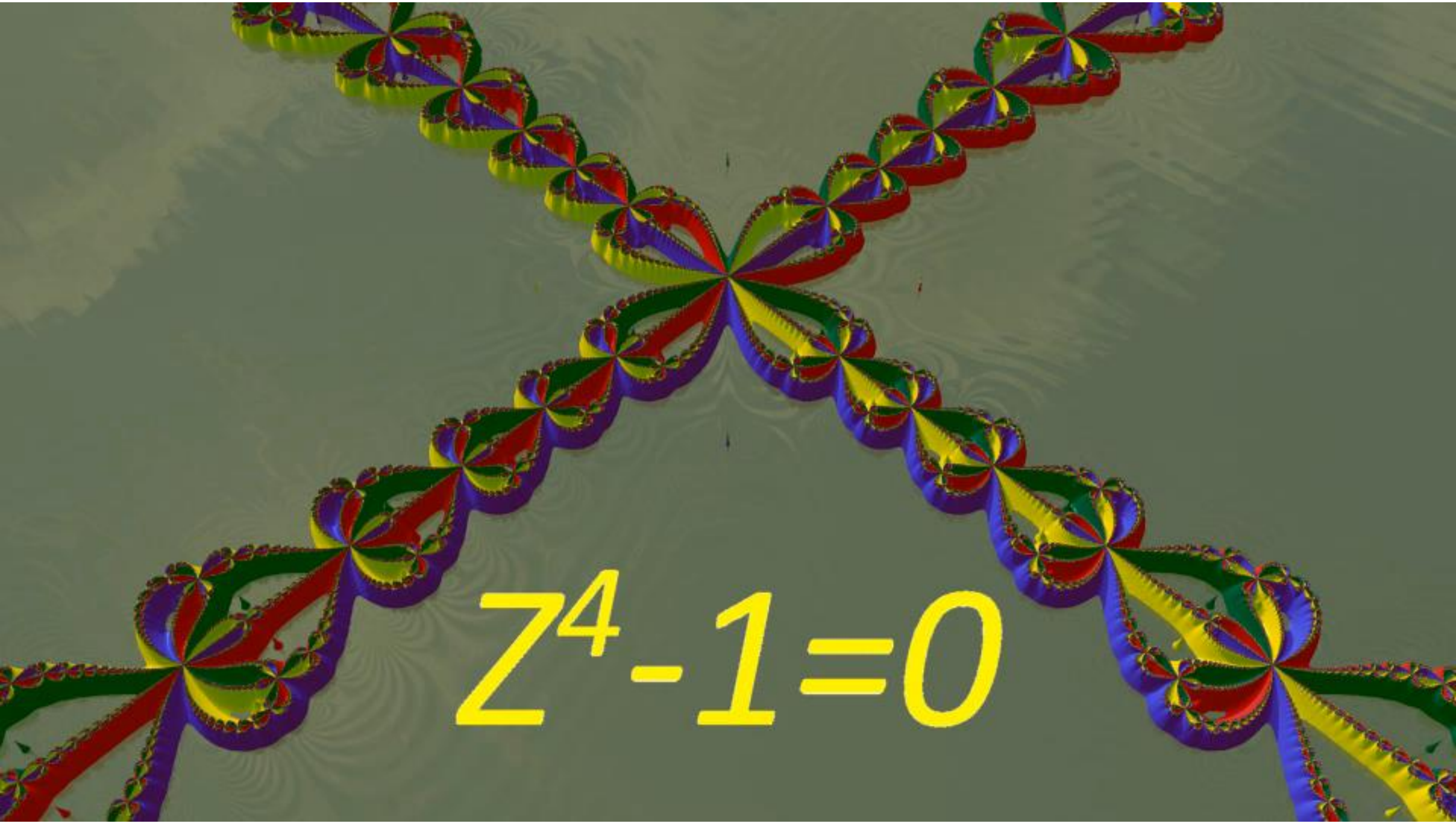

Mandelbrot
flight #2

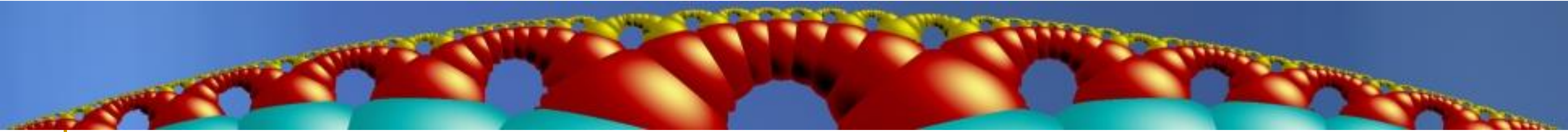









Z4-1=0

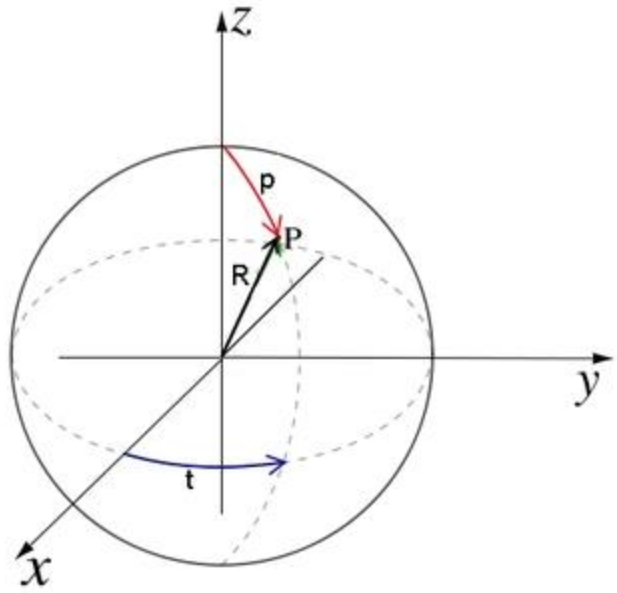


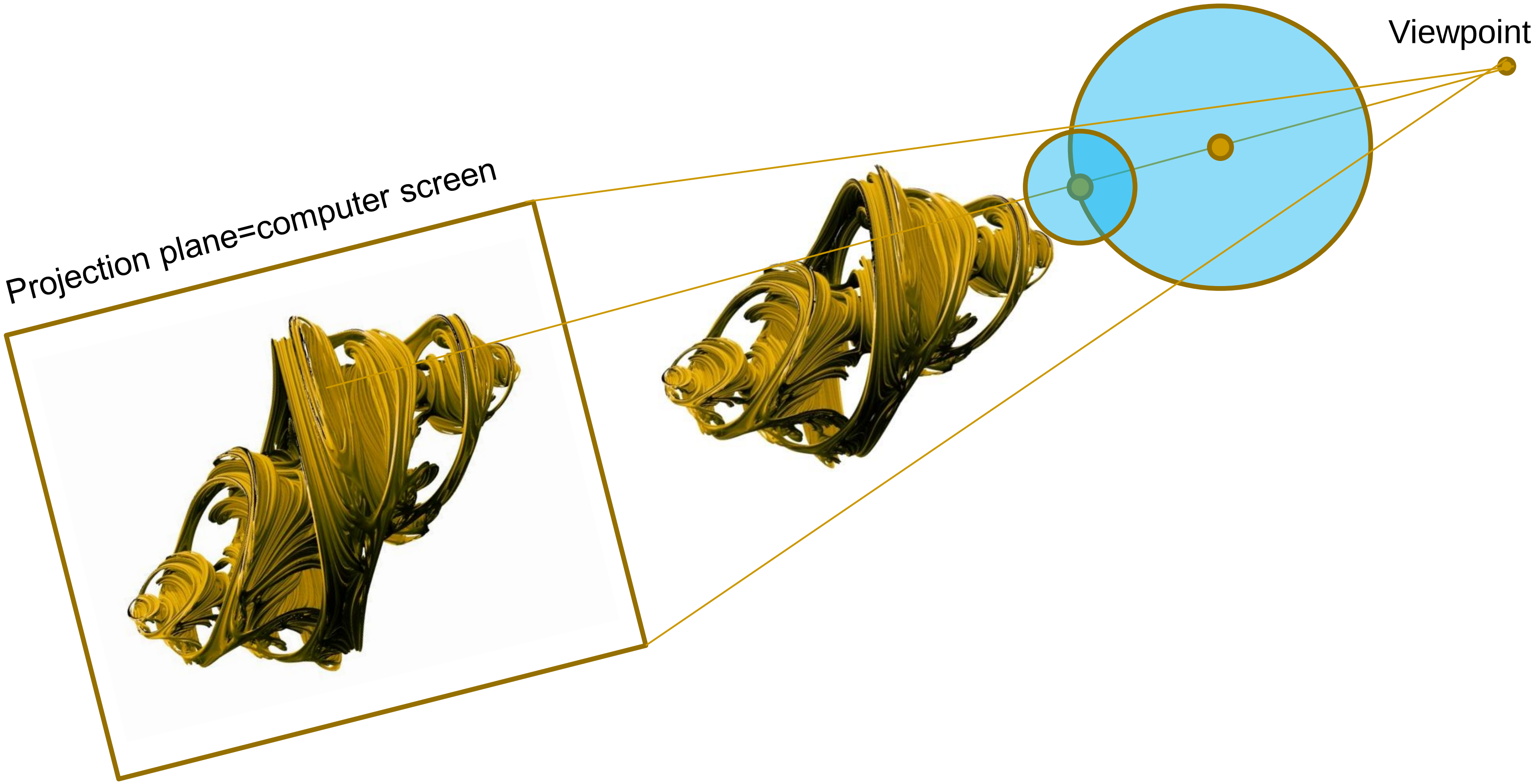
Use a point's spherical coordinates, and iteratively square the distance to the origin and double both angles.

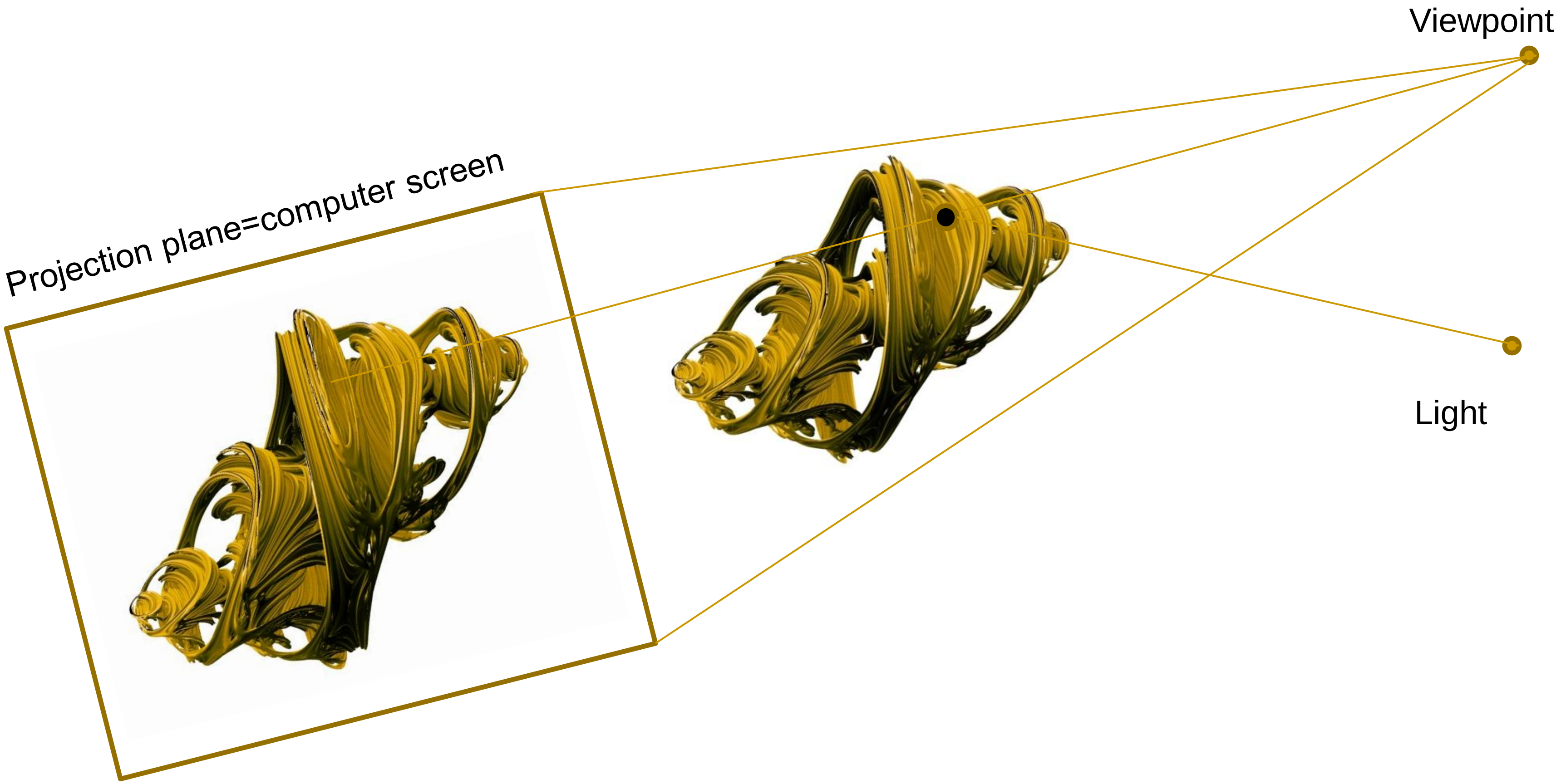
$$x \to R^2 \cos 2\theta \ \cos 2\varphi + c_x$$
$$y \to R^2 \cos 2\theta \ \sin 2\varphi + c_y$$
$$z \to R^2 \sin 2\theta + c_z$$

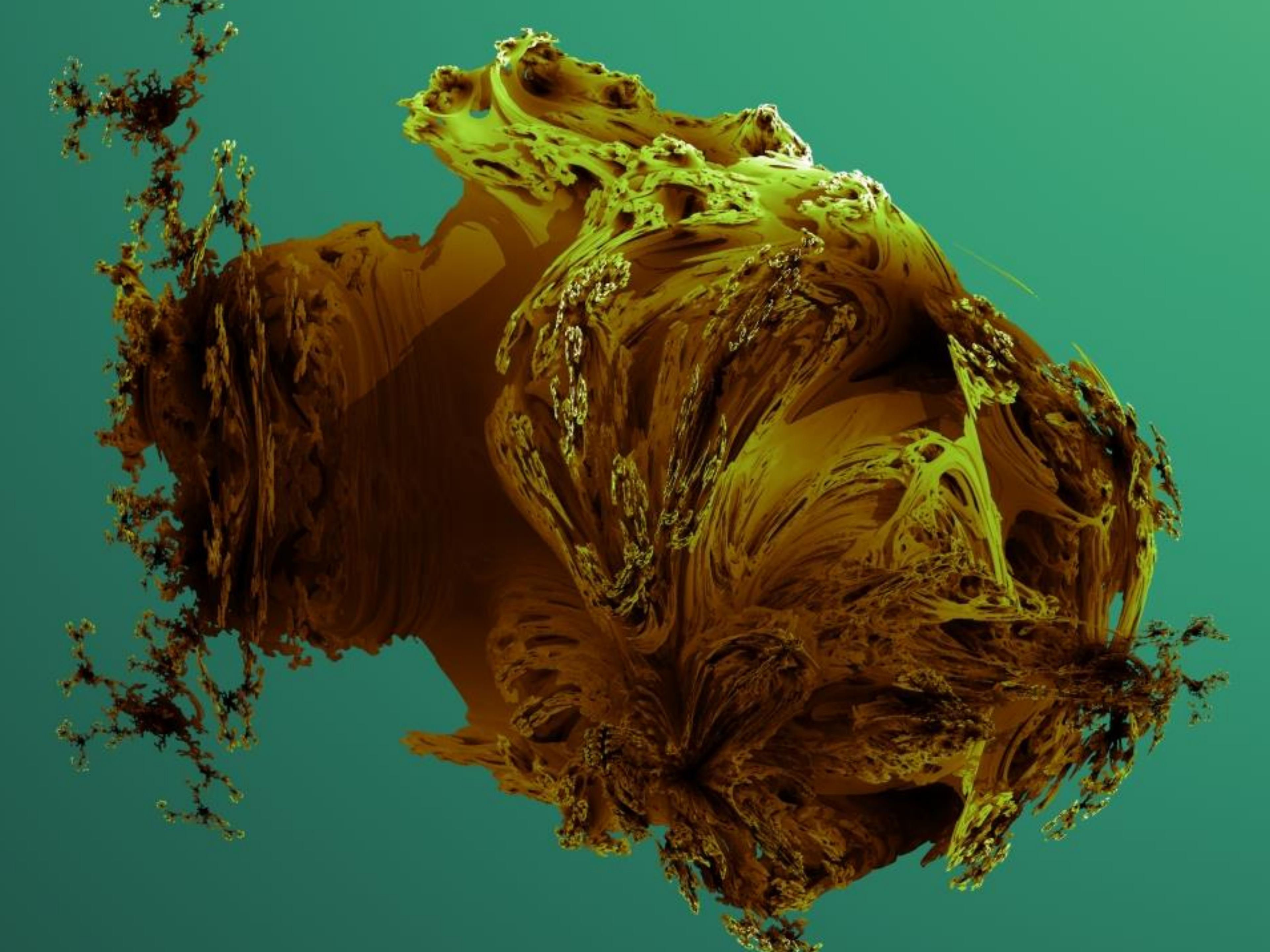
Distance estimate :

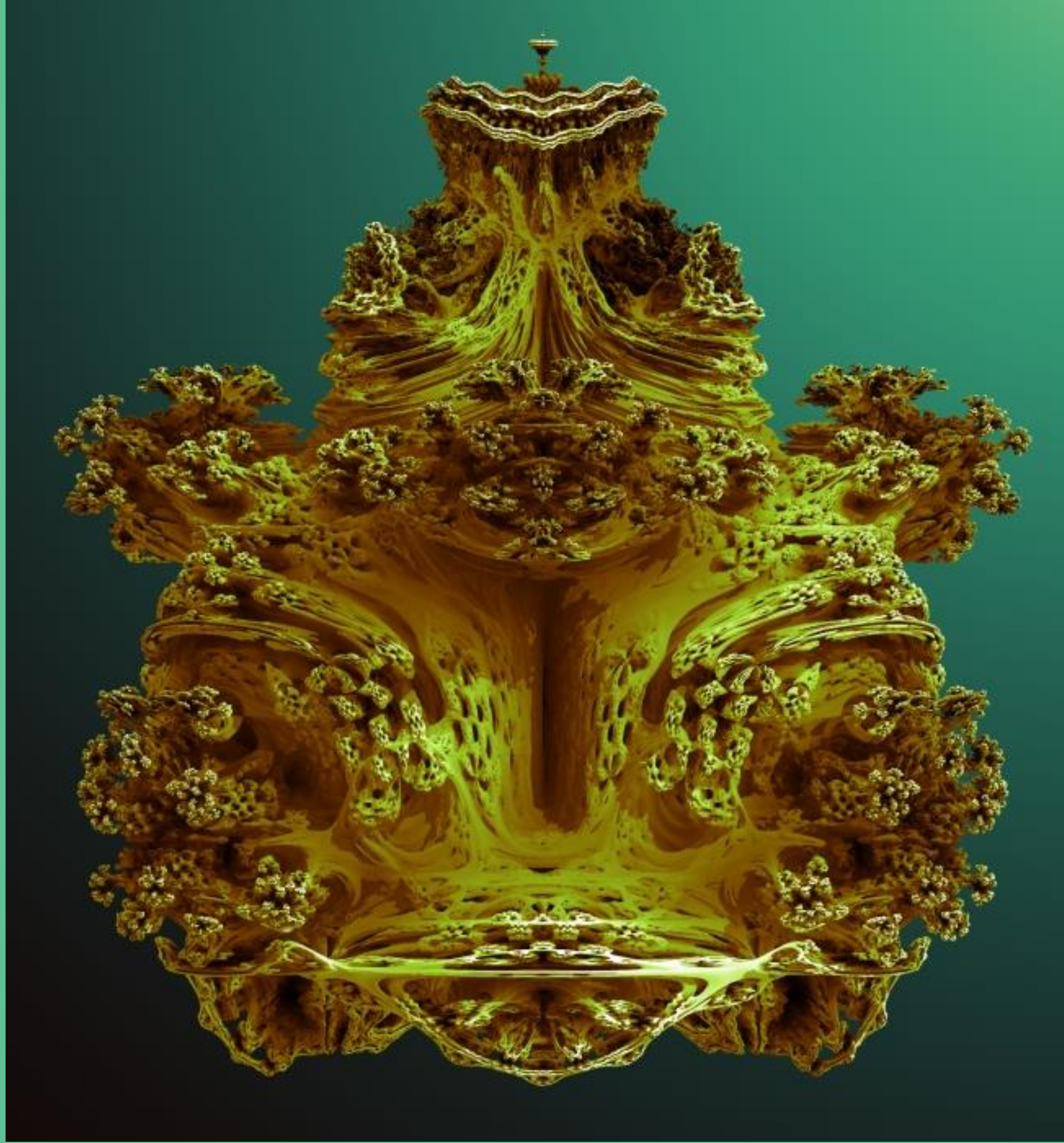
$$d < \frac{ln(R)R}{2dR}$$

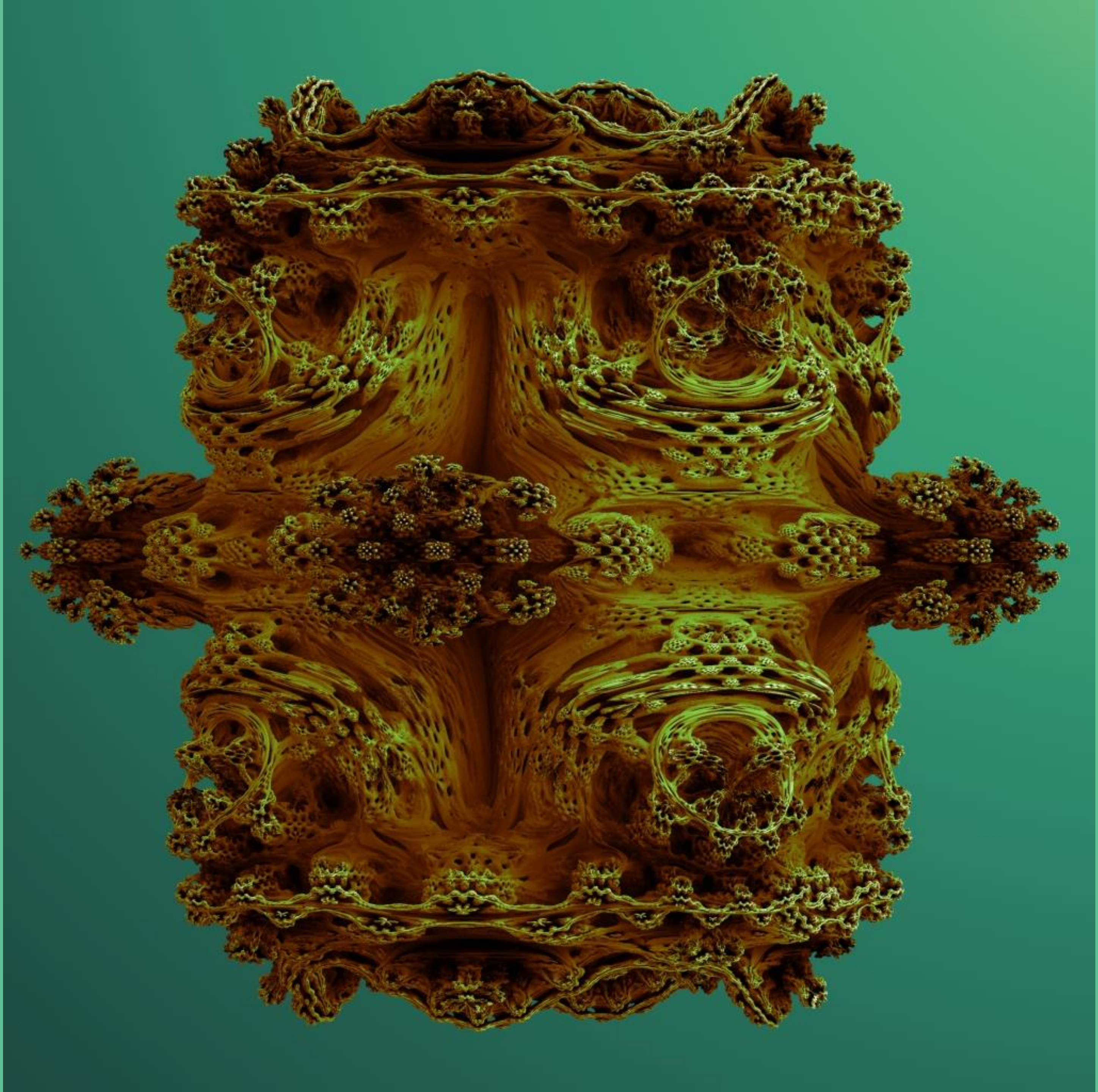



Viewpoint
Projection plane=computer screen

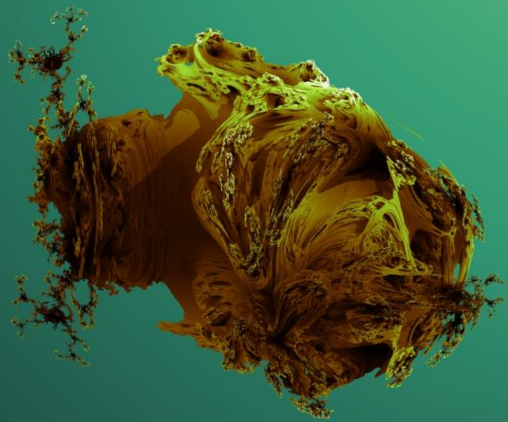

Viewpoint
Projection plane=computer screen
Light

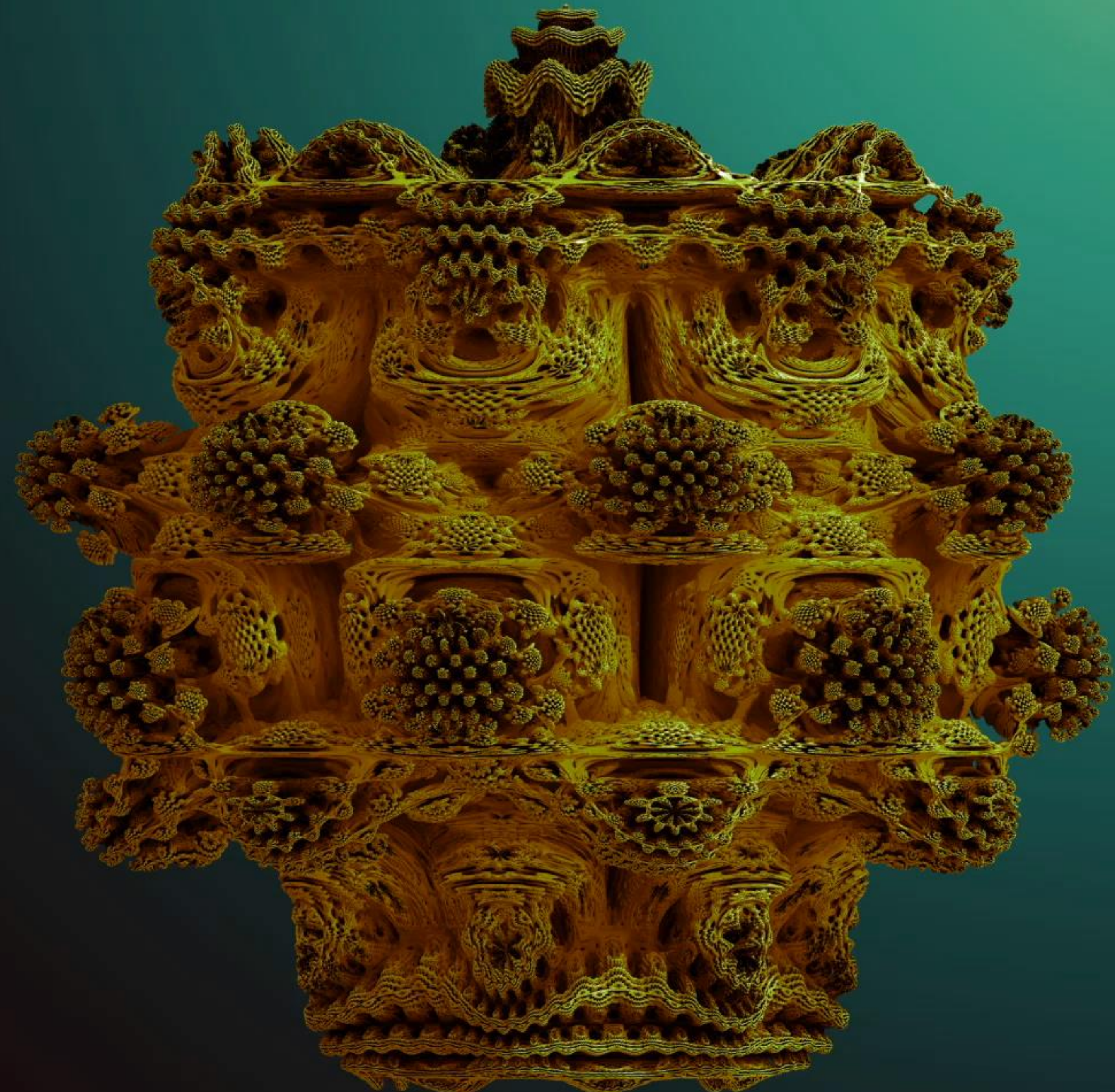




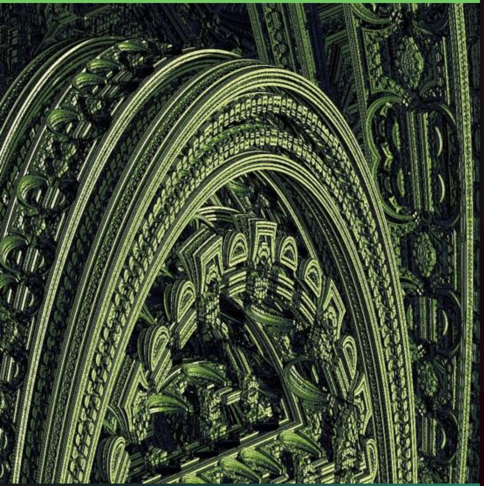


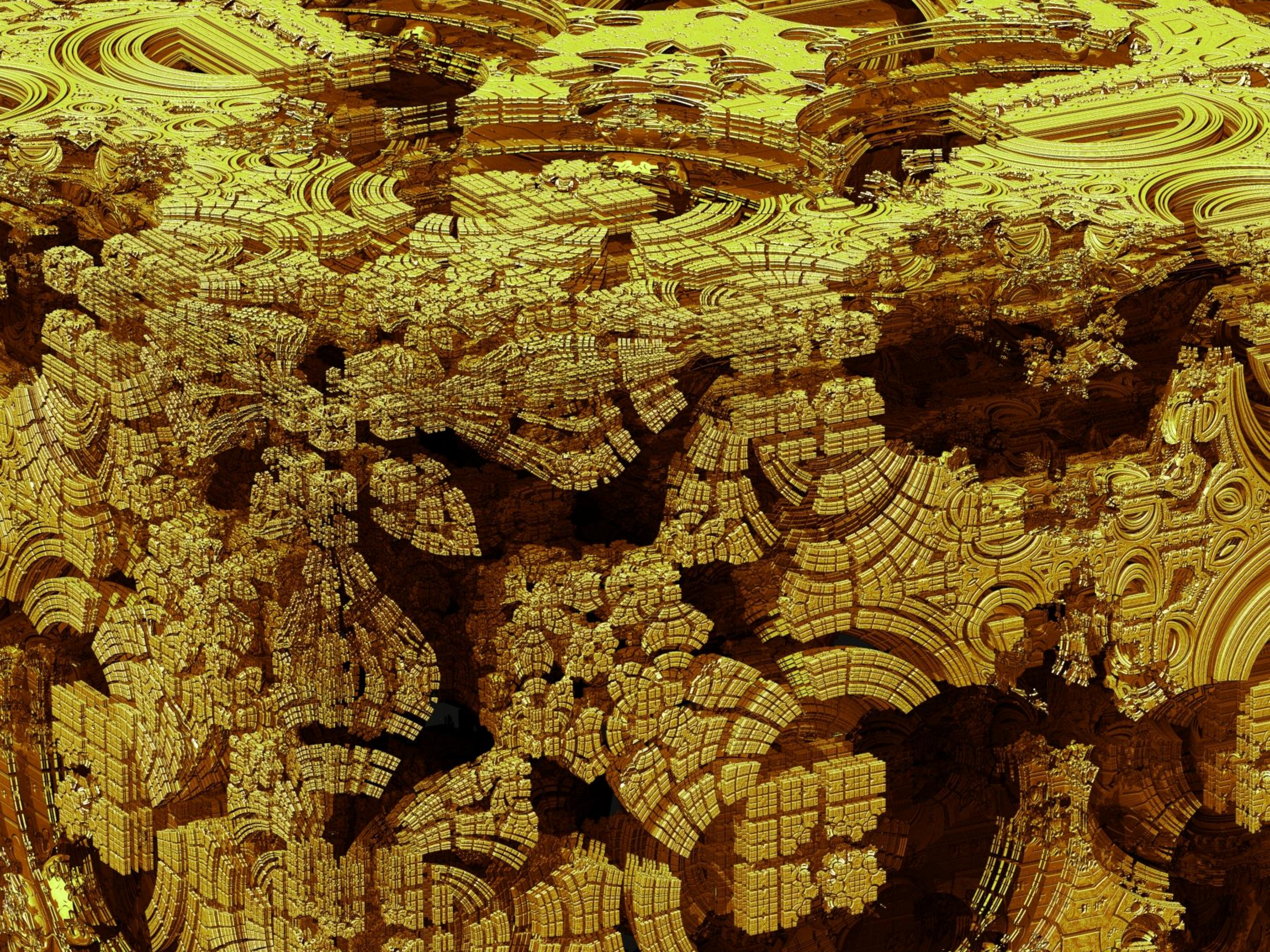
The 'Mandelbulb'

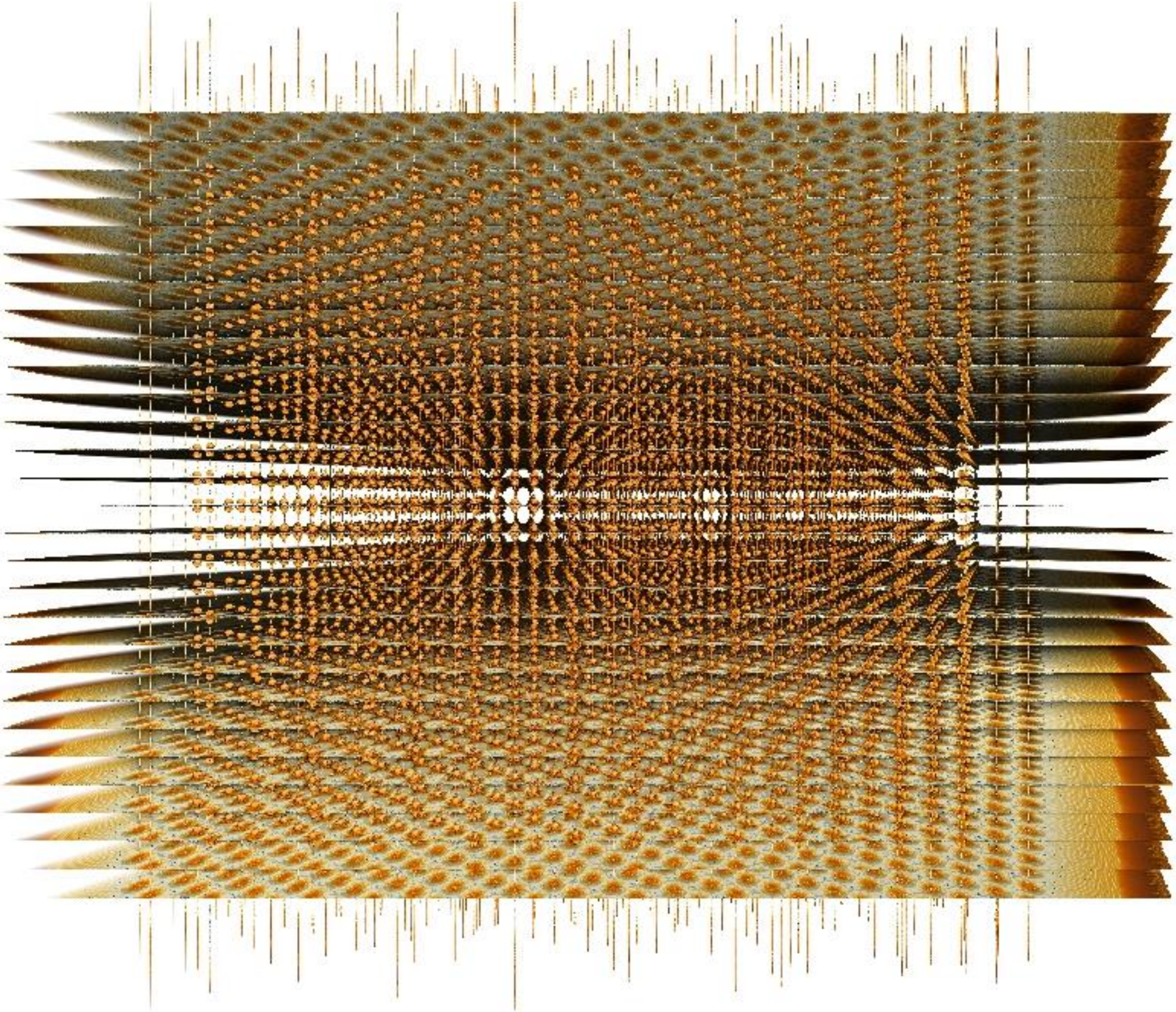




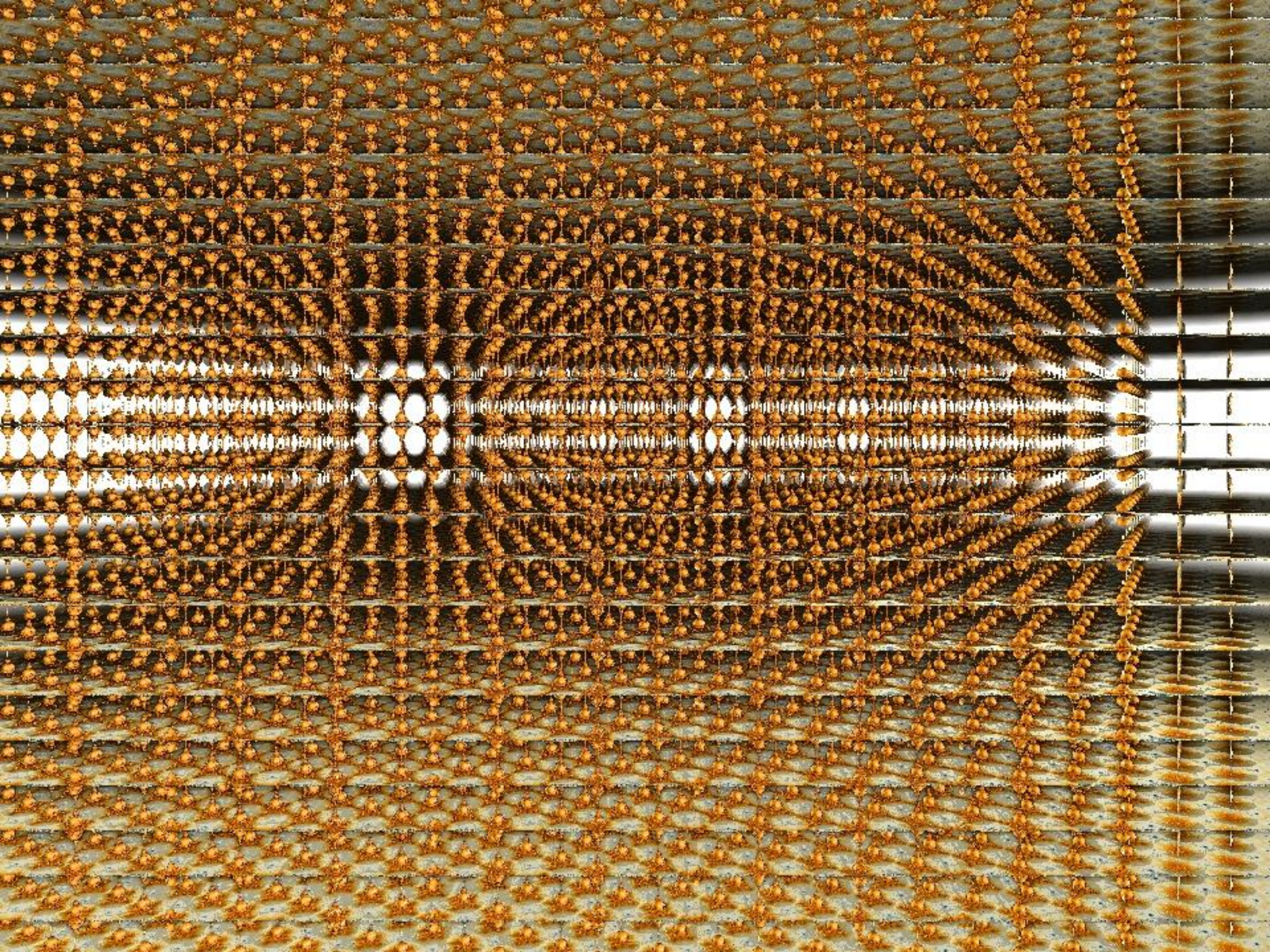
The 'Mandelbox'

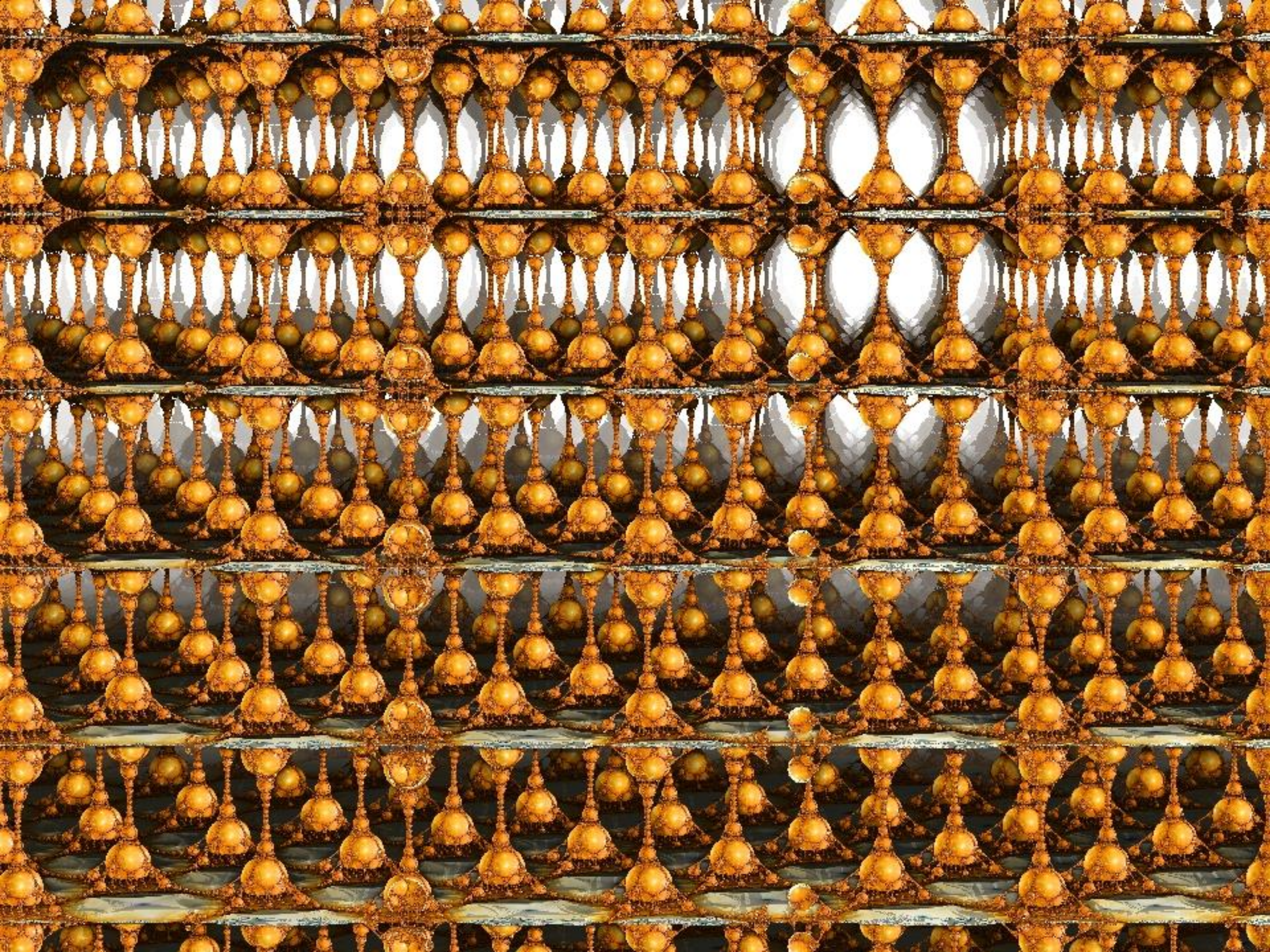


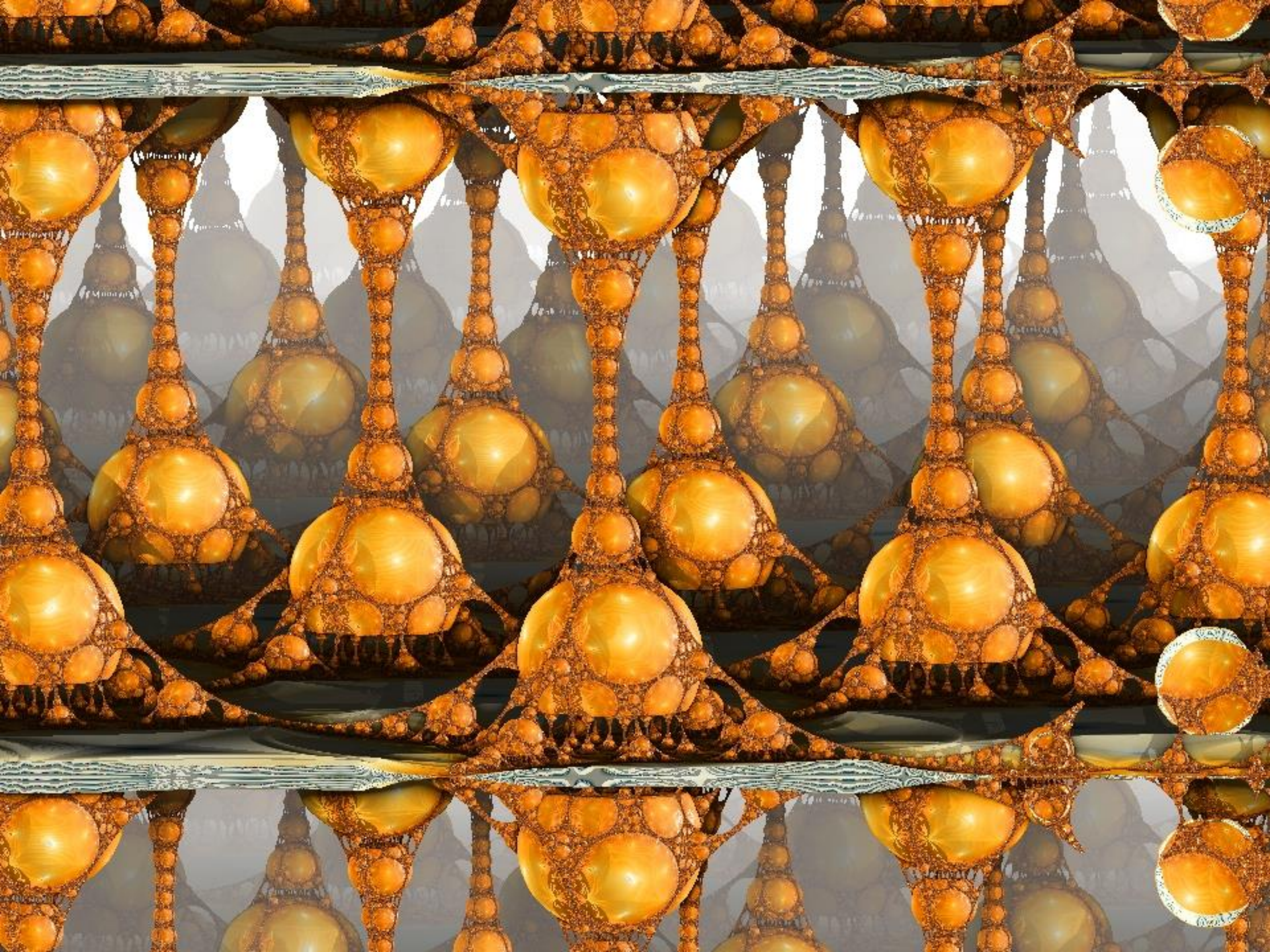


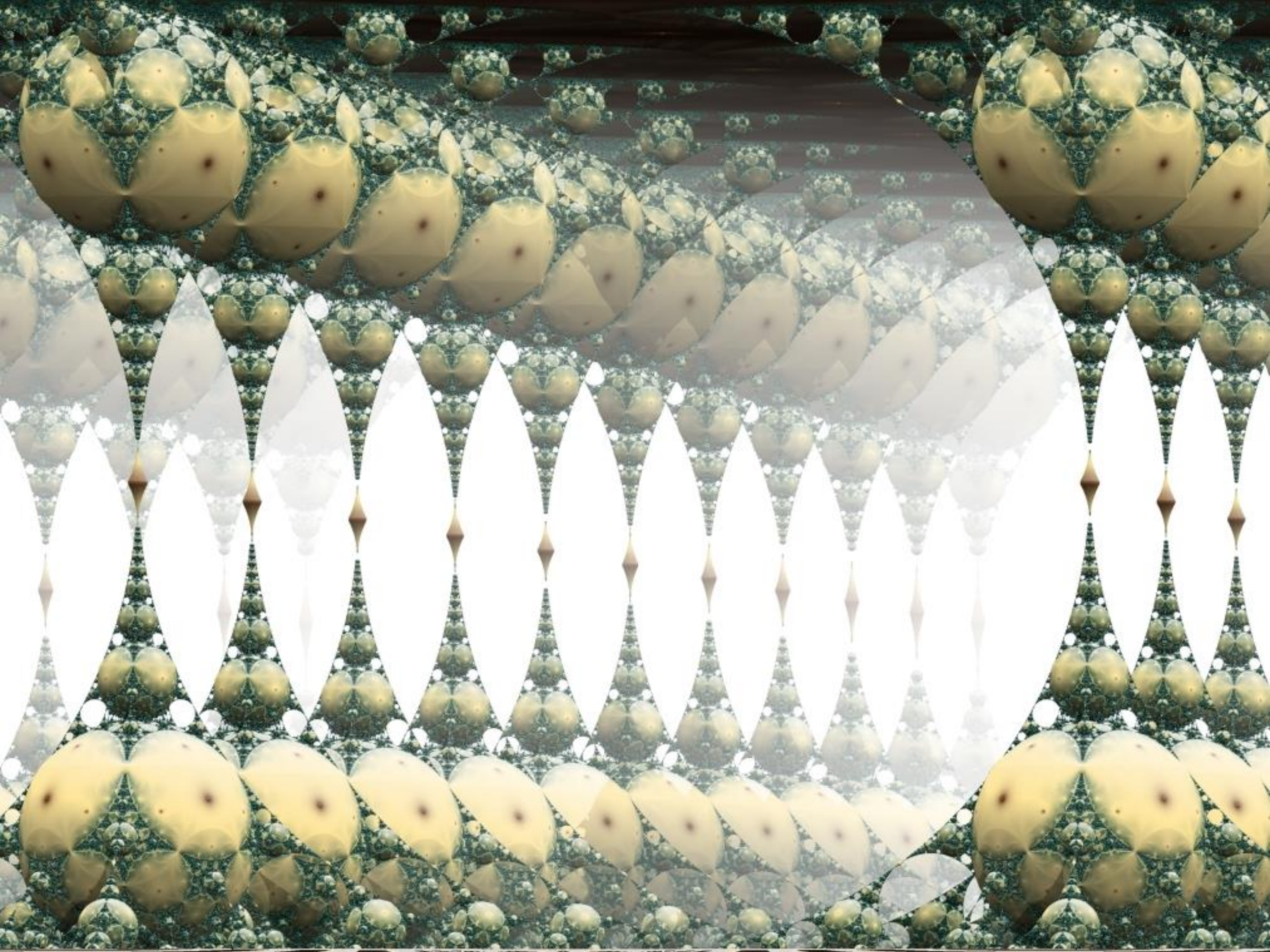


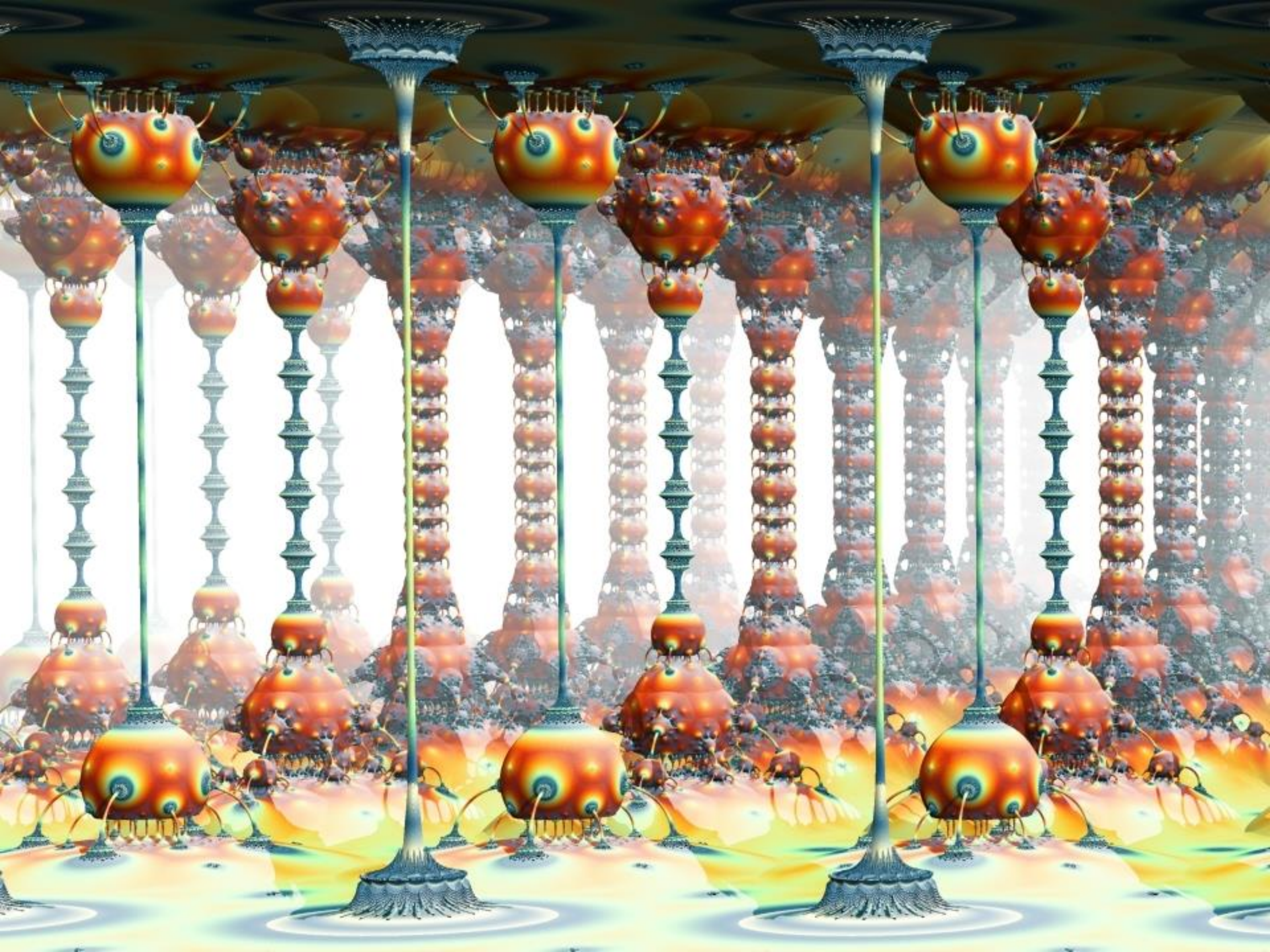
Hybrid Mandelbulb-Mandelbox

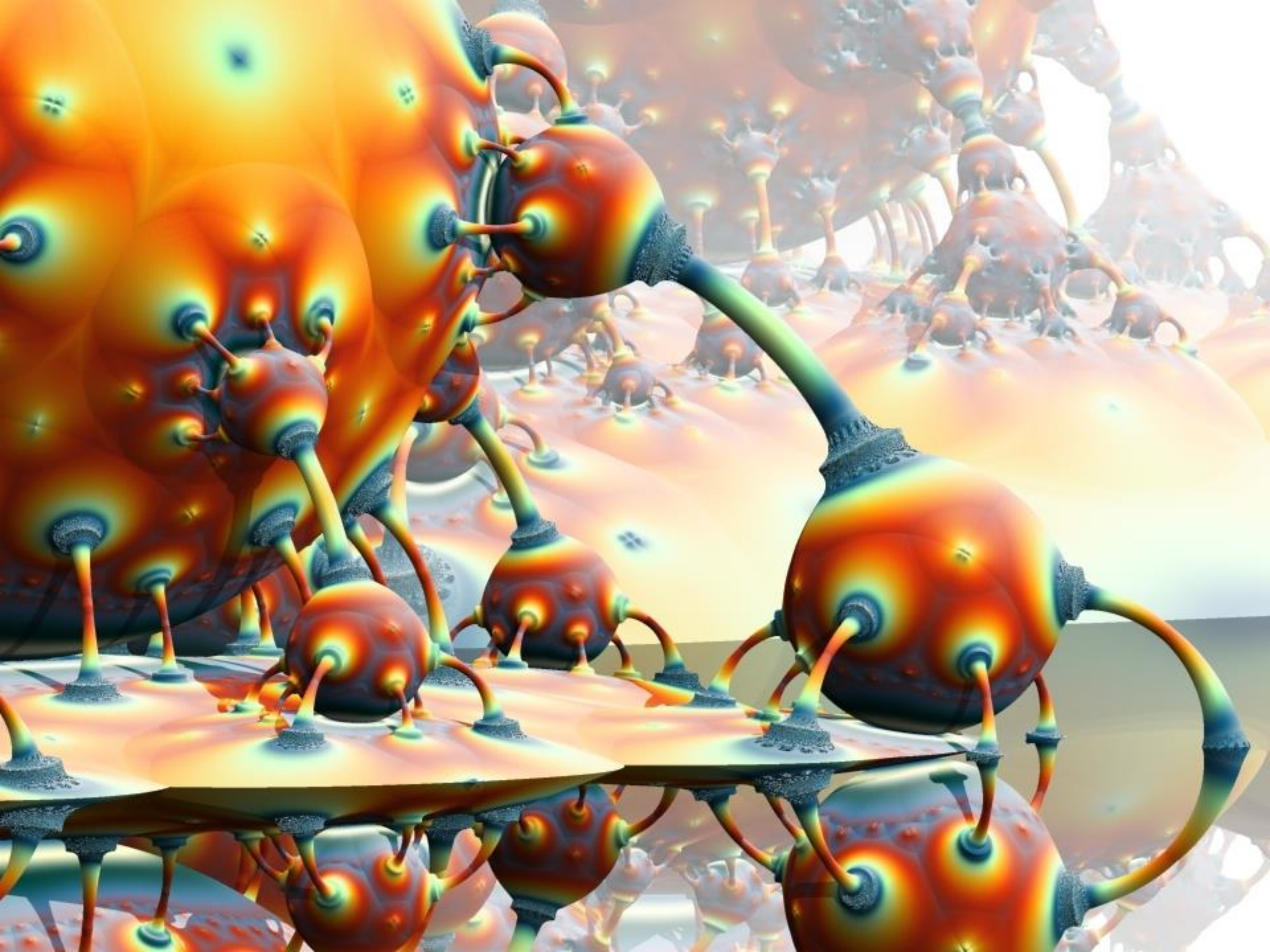


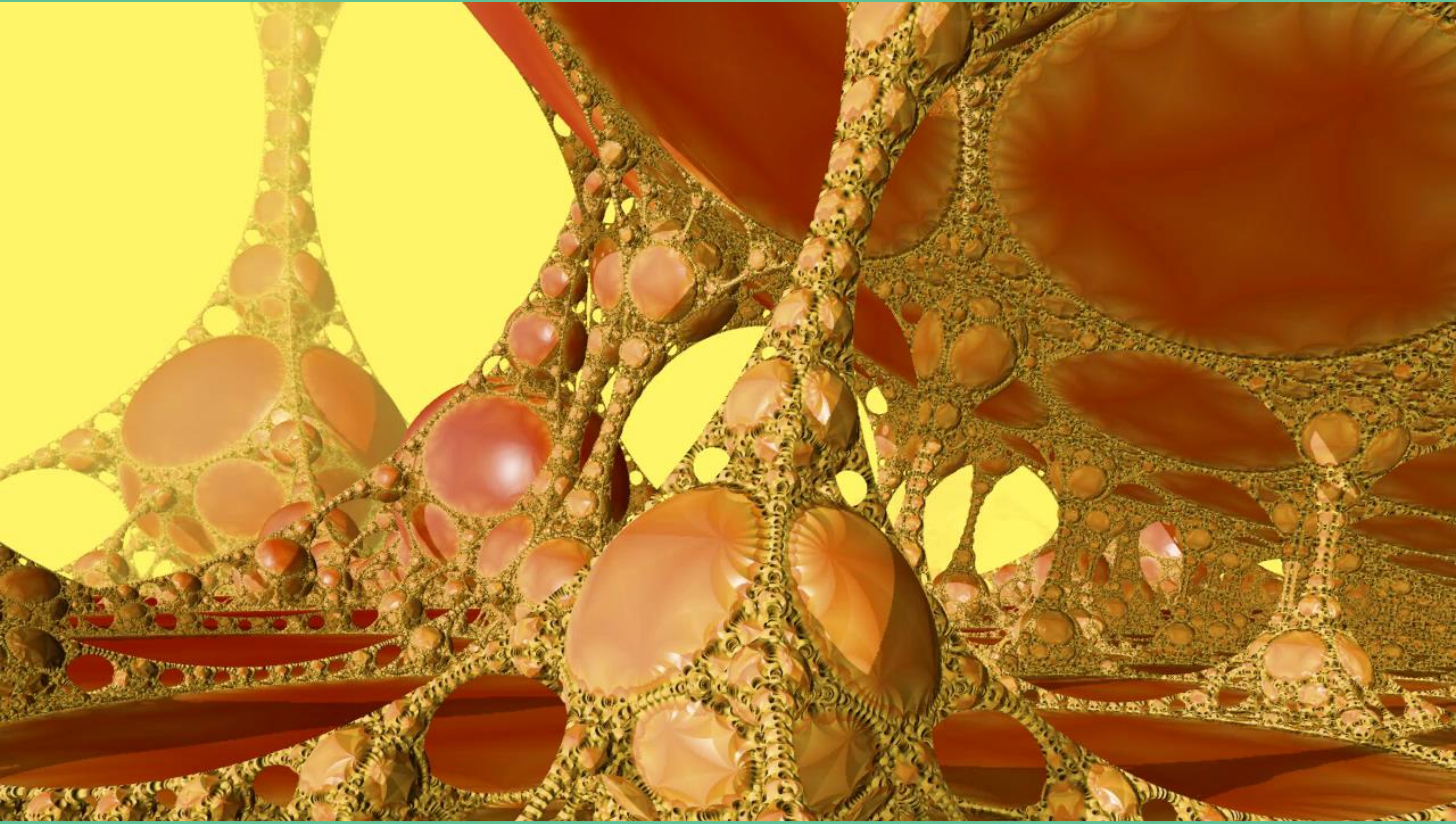






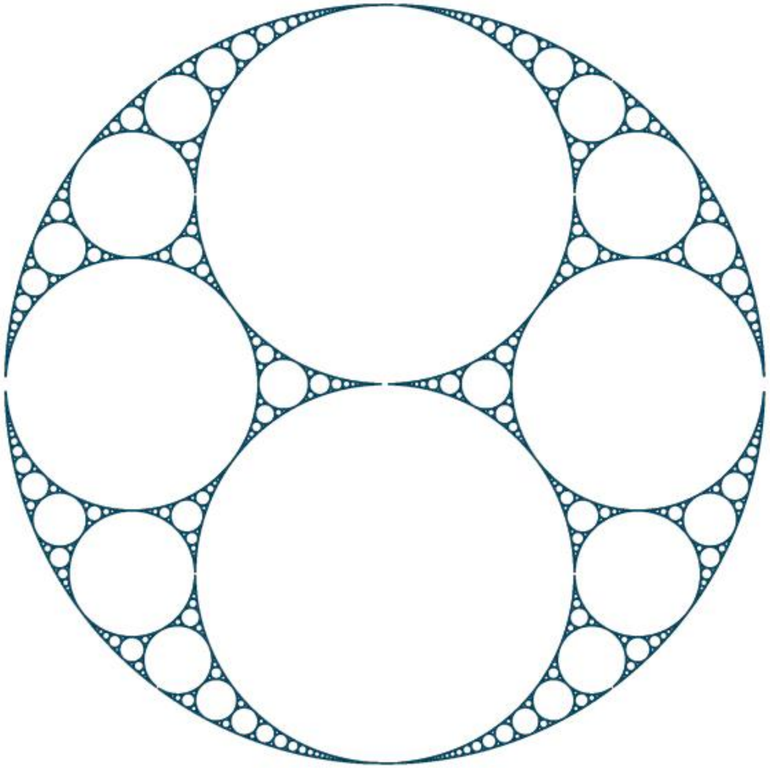






$f(z) = \frac{z}{-2iz+1}$ or $\begin{vmatrix} 1 & 0 \\ -2i & 1 \end{vmatrix}$

$g(z) = \frac{(1-i)z+1}{z+(1+i)}$ or $\begin{vmatrix} 1-i & 1 \\ 1 & 1+i \end{vmatrix}$



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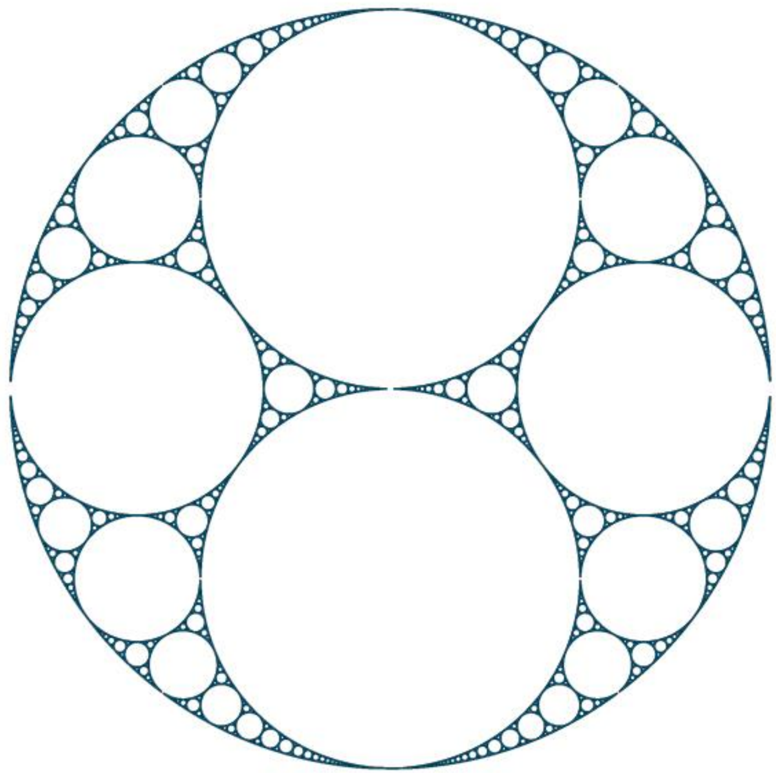
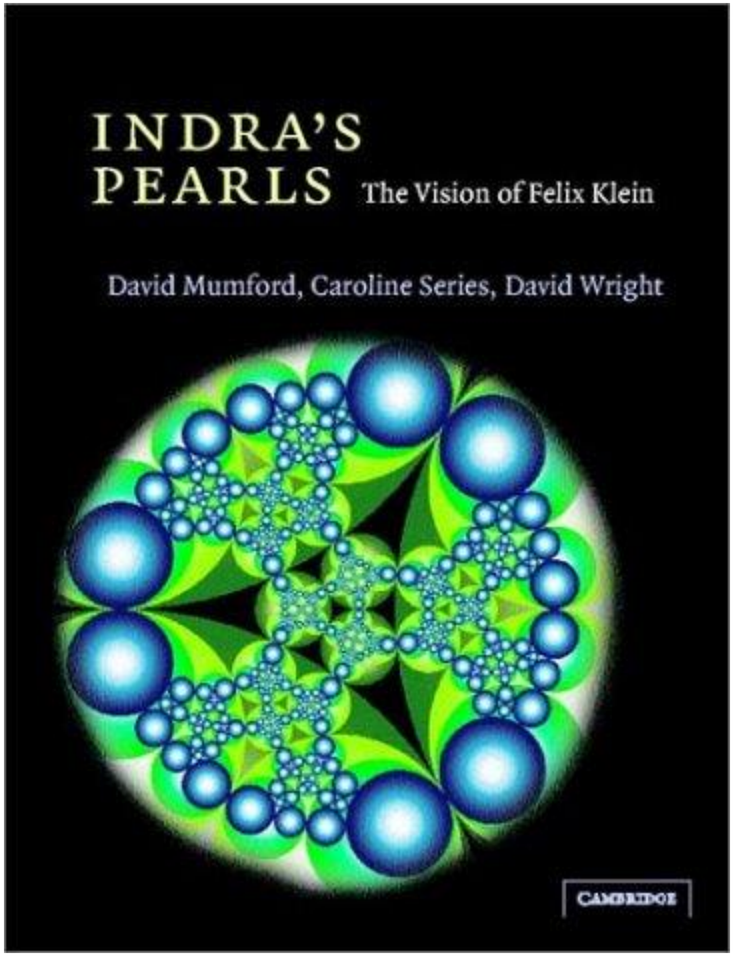
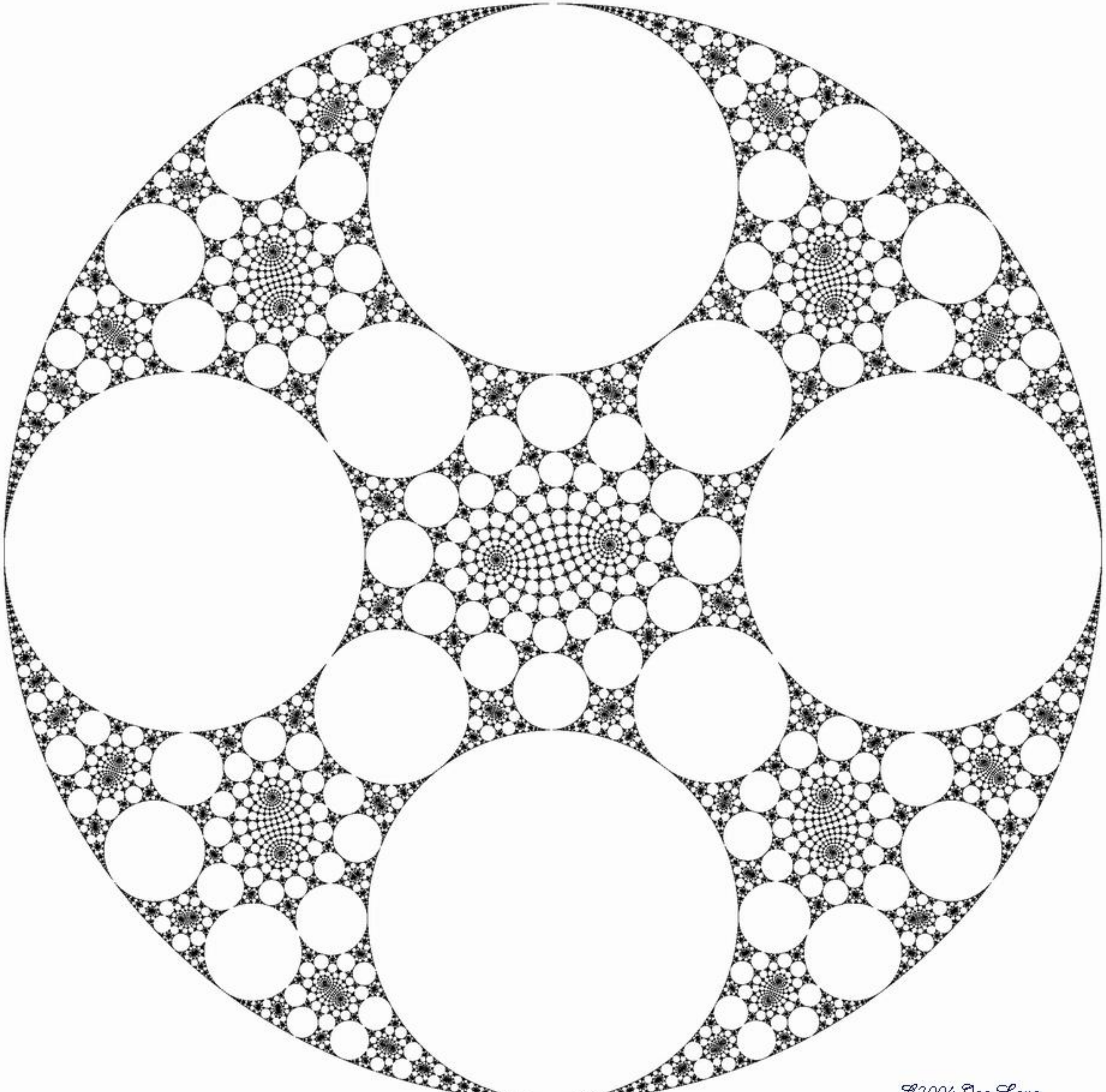


Indra's Pearls

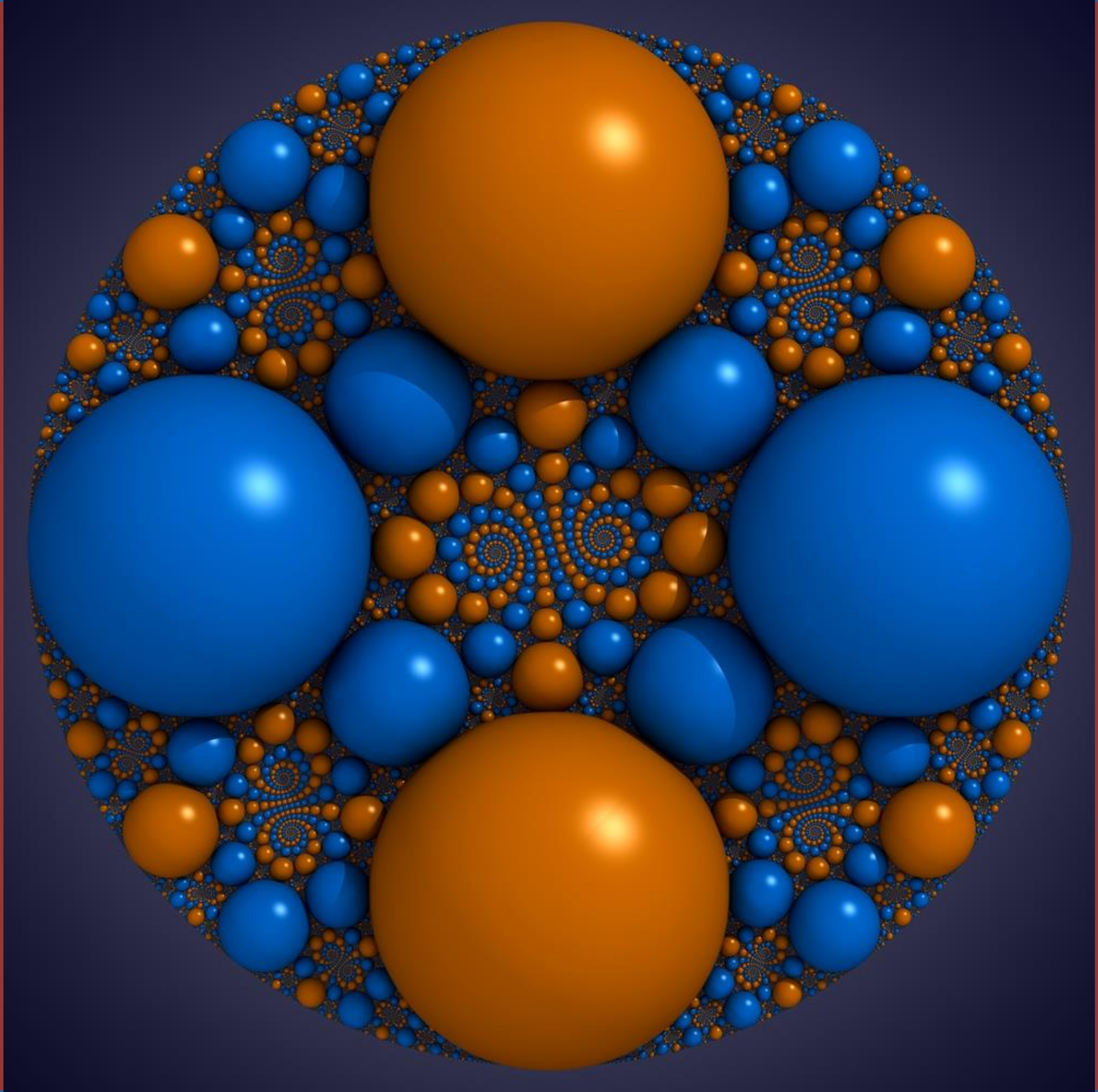
- Published 2002, with accessible math!
- Limit sets of groups of Moebius transformations.

- The book explains a generic code for the images.

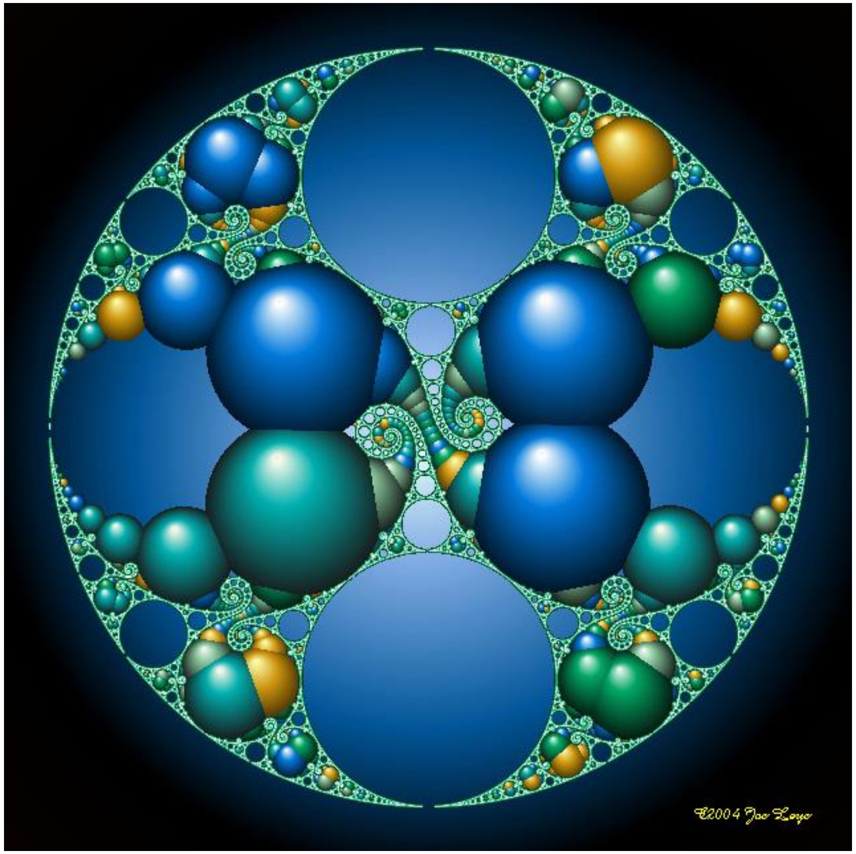


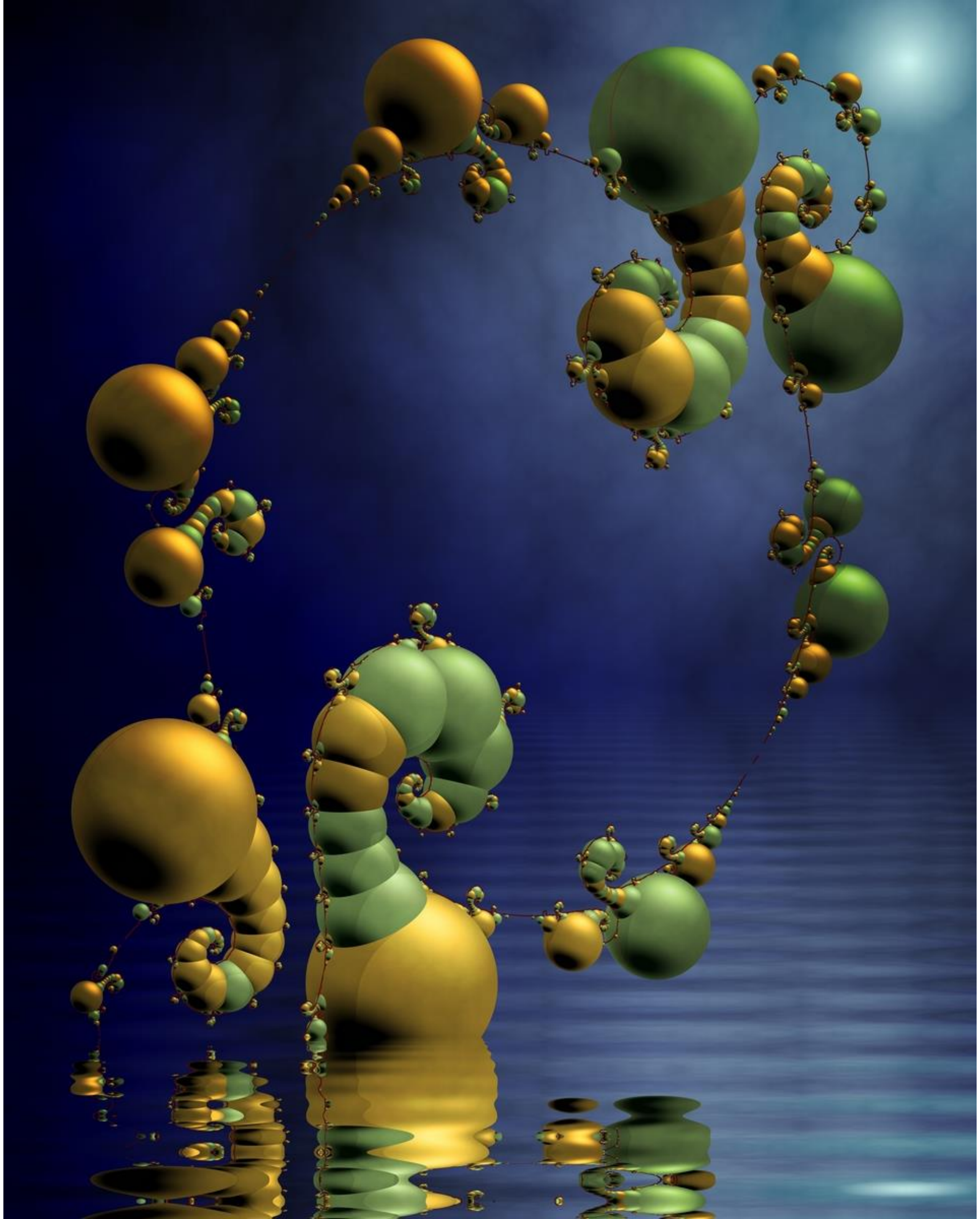


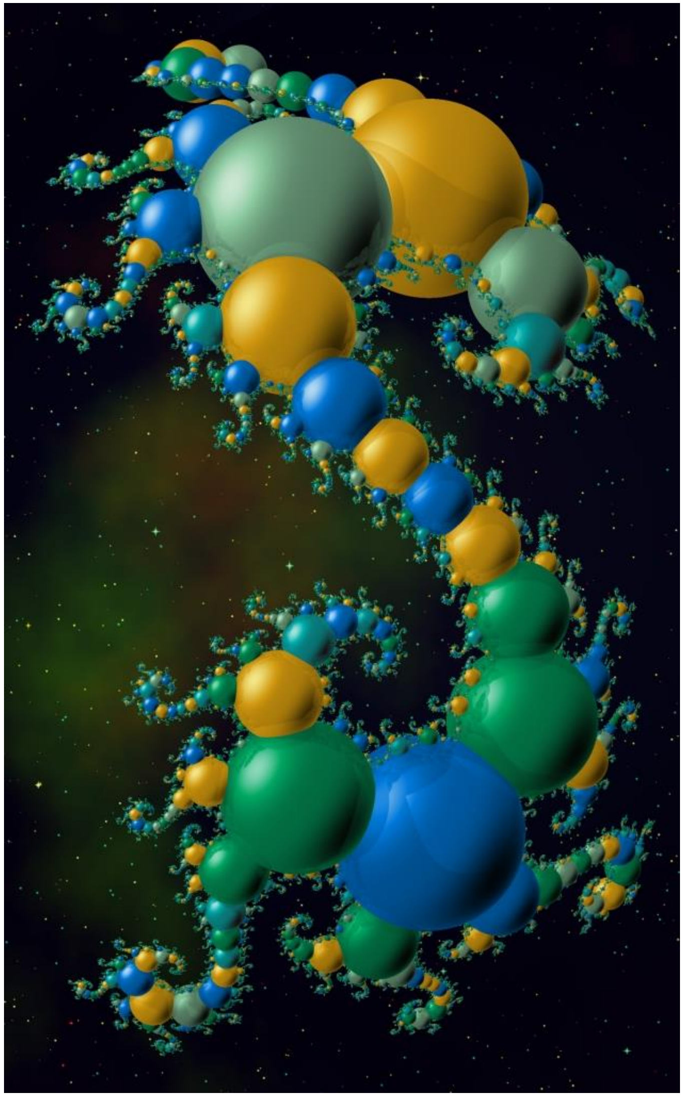
©2004 Jos Leys

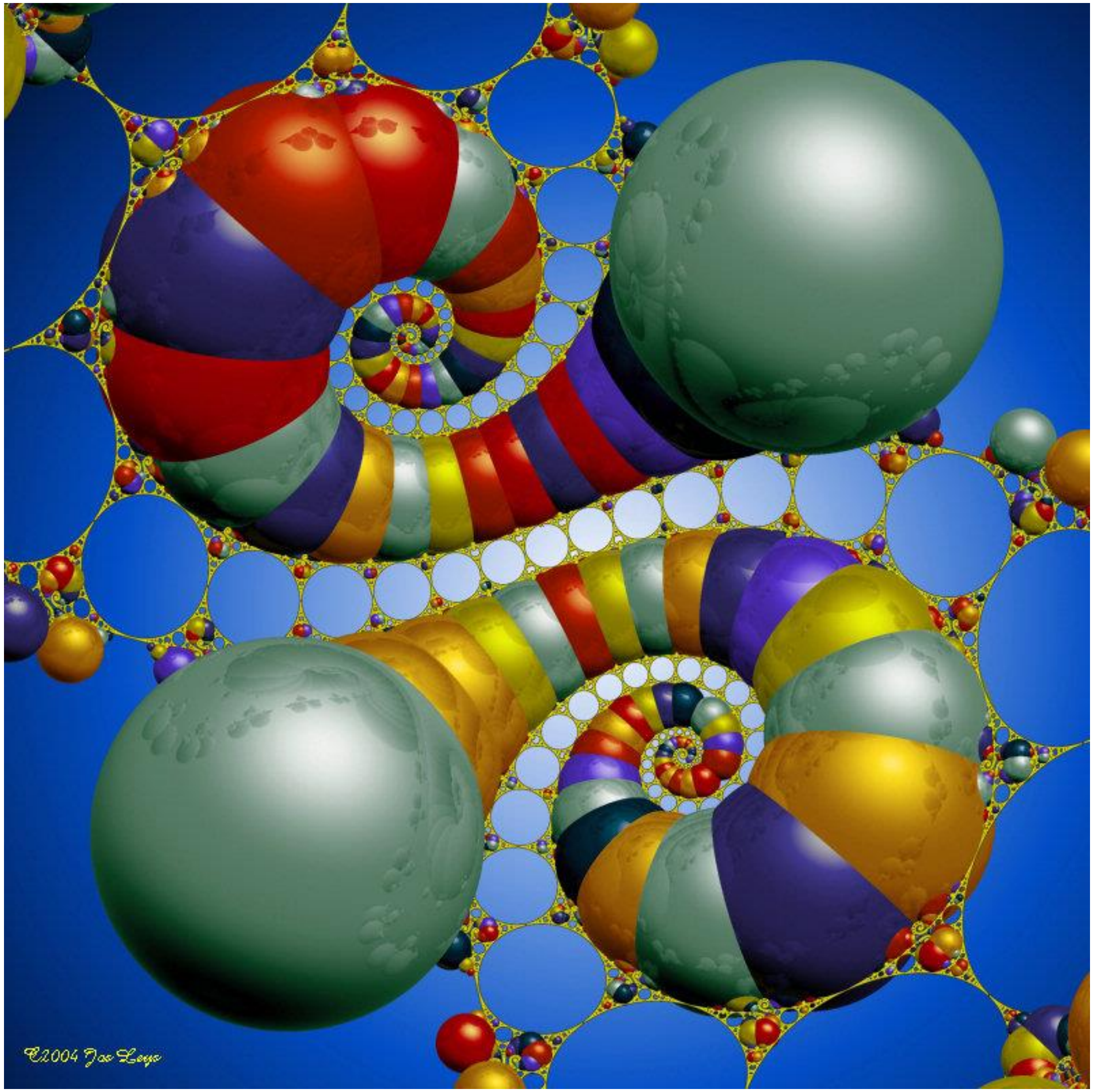


…spheres instead of circles…





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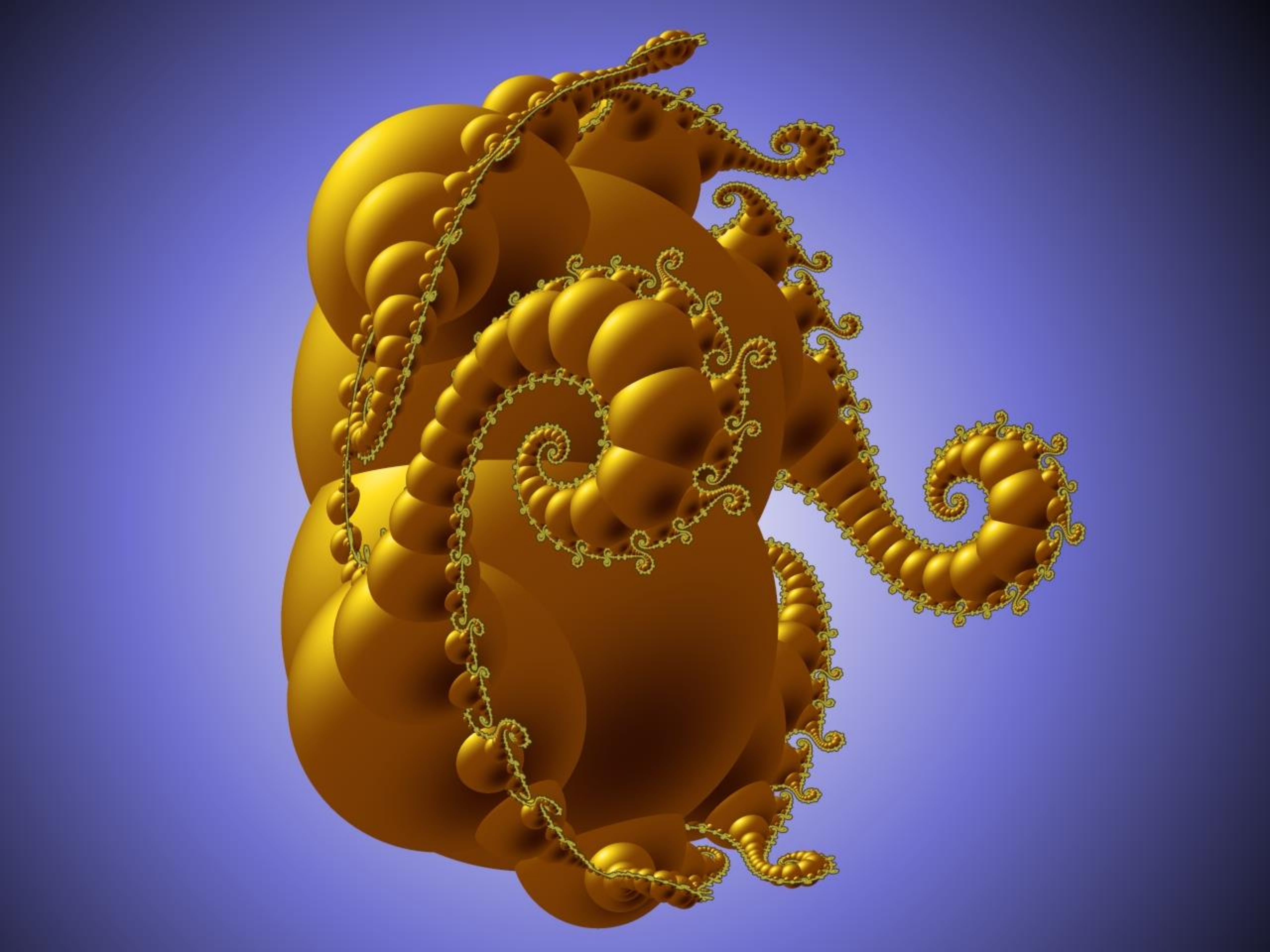


AN EXTENSION OF THE MASKIT SLICE FOR 4-DIMENSIONAL KLEINIAN GROUPS

YOSHIAKI ARAKI AND KENTARO ITO

Abstract. Let Γ be a 3-dimensional Kleinian punctured torus group with accidental parabolic transformations. The deformation space of Γ in the group of Möbius transformations on the 2-sphere is well-known as the Maskit slice $\mathcal{M}_{1,1}$ of punctured torus groups. In this paper, we study deformations Γ' of Γ in the group of Möbius transformations on the 3-sphere such that Γ' does not contain screw parabolic transformations. We will show that the space of the deformations is realized as a domain of 3-space $\mathbb{R}^3$, which contains the Maskit slice $\mathcal{M}_{1,1}$ as a slice through a plane. Furthermore, we will show that the space also contains the Maskit slice $\mathcal{M}_{0,4}$ of fourth-punctured sphere groups as a slice through another plane. Some of another slices of the space will be also studied.

1. Introduction

Let Γ be a 3-dimensional Kleinian once-punctured (or simply punctured)

A true 3D version...

- Moebius transformations work in 3D also, provided we use *quaternions* instead of complex numbers:

$$q = x + iy + jz + kw$$

with

$$i^2 = -1,\ j^2 = -1,\ k^2 = -1,\ ij = k,\ jk = i,\ ki = j,\ ijk = -1$$

- Express a point in space as a quaternion whose fourth element is zero :

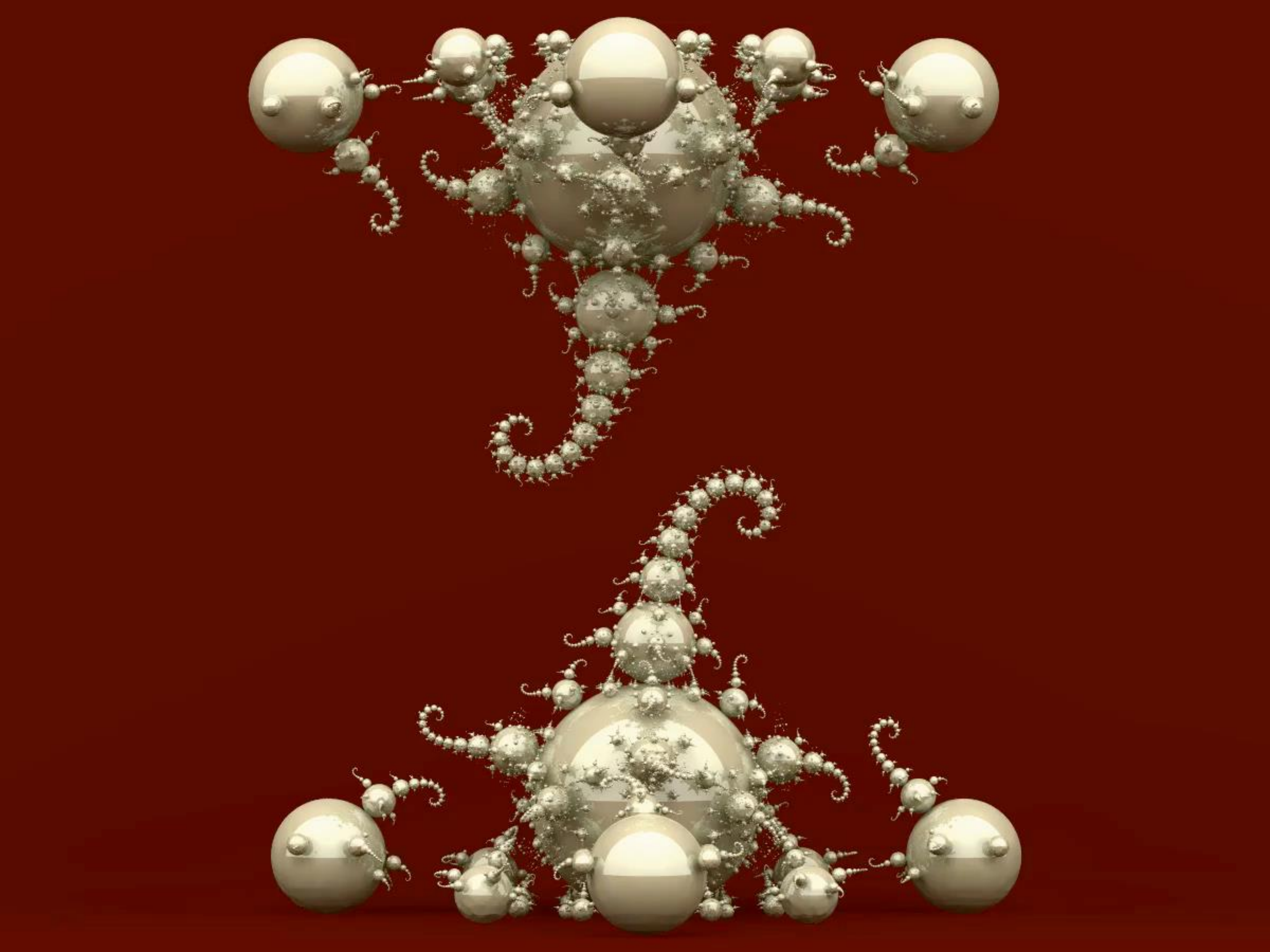
$$P = x + iy + jz + k.0$$

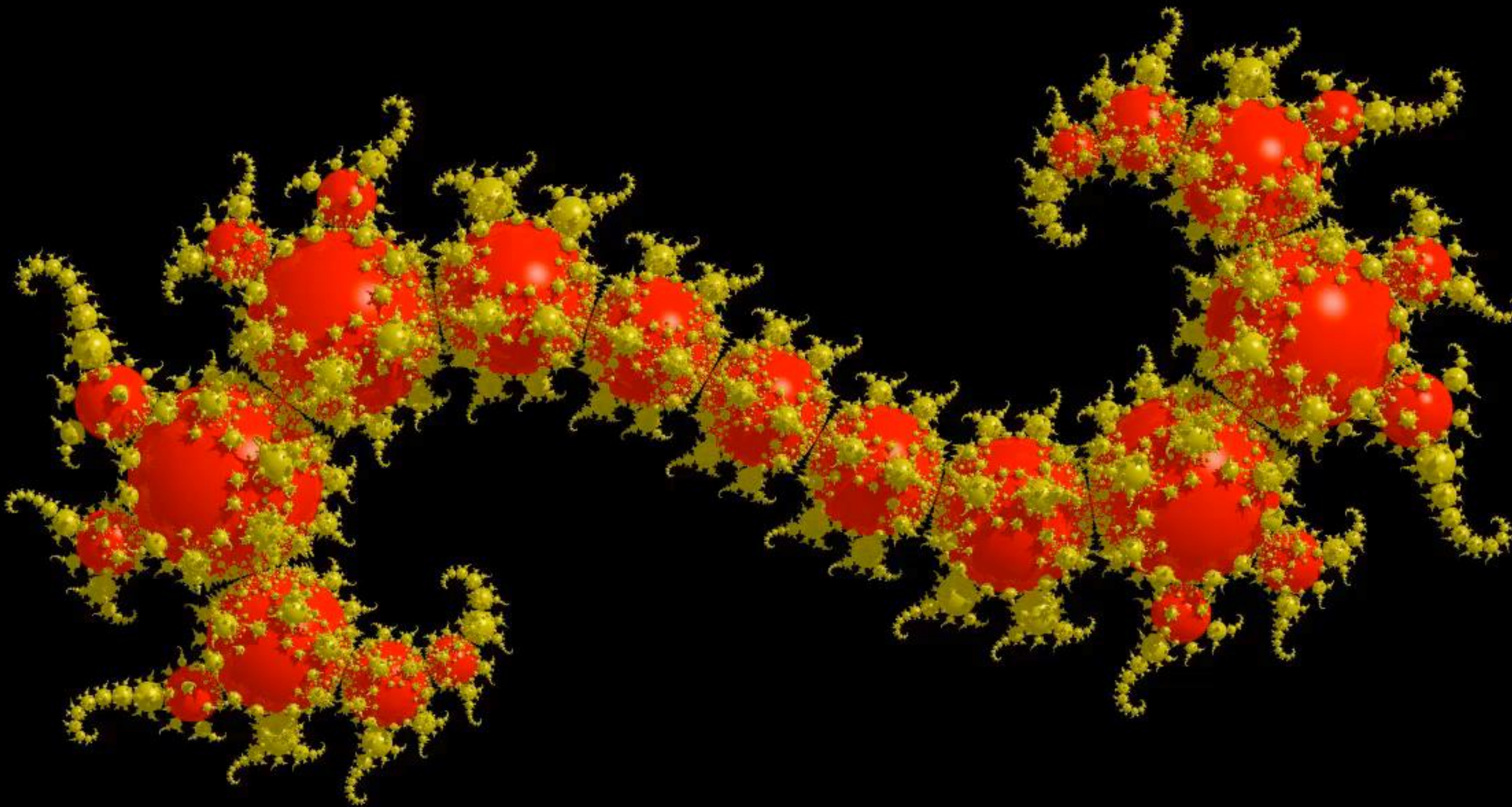
- Now take three quaternionic Moebius transforms: two translations and a combination of a translation and a sphere inversion.

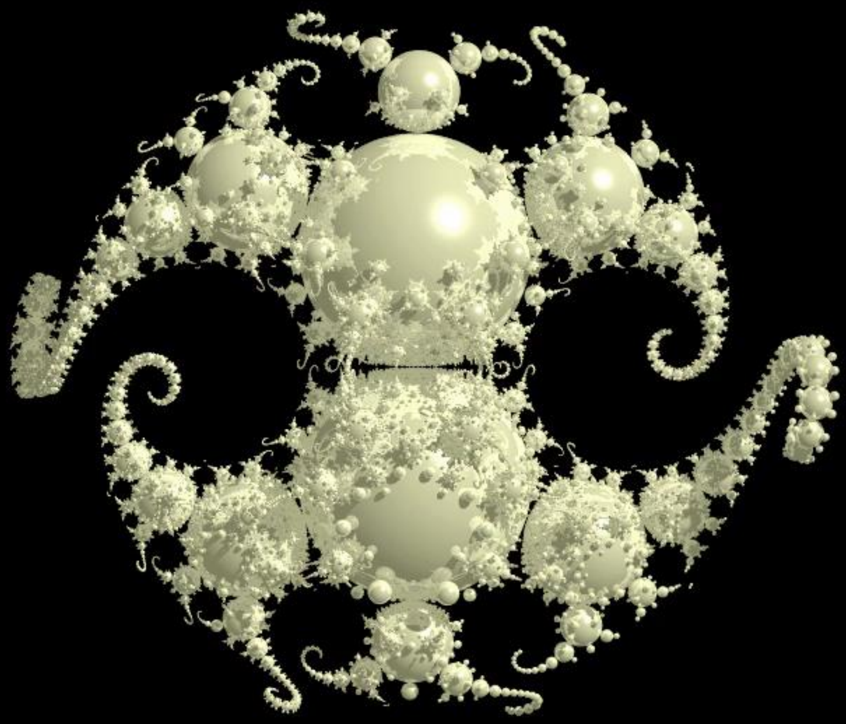
$$a = \begin{pmatrix} t & -i \\ -i & 0 \end{pmatrix}, b = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}, c = \begin{pmatrix} 1 & 2j \\ 0 & 1 \end{pmatrix}$$

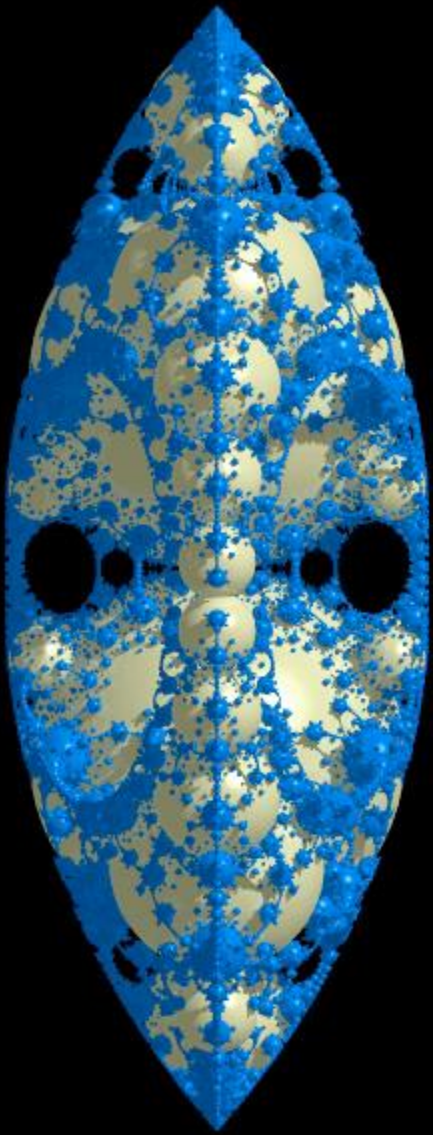
...and their inverses.

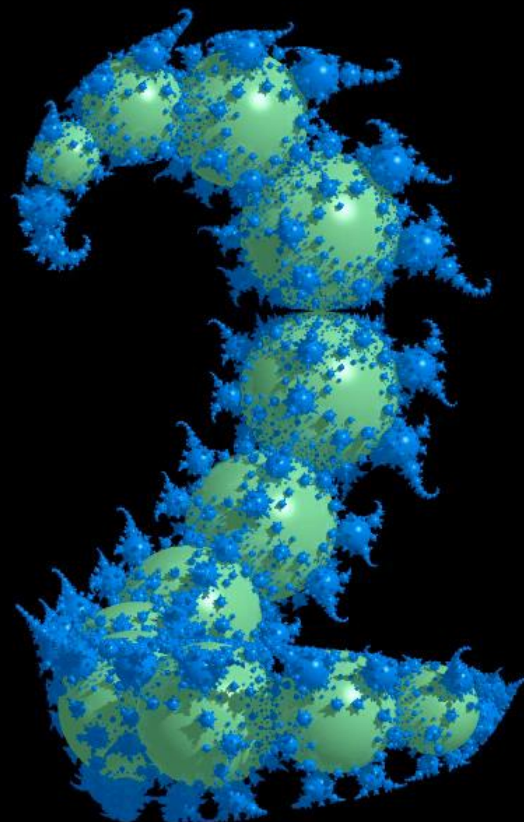
- Choose a proper value for t..

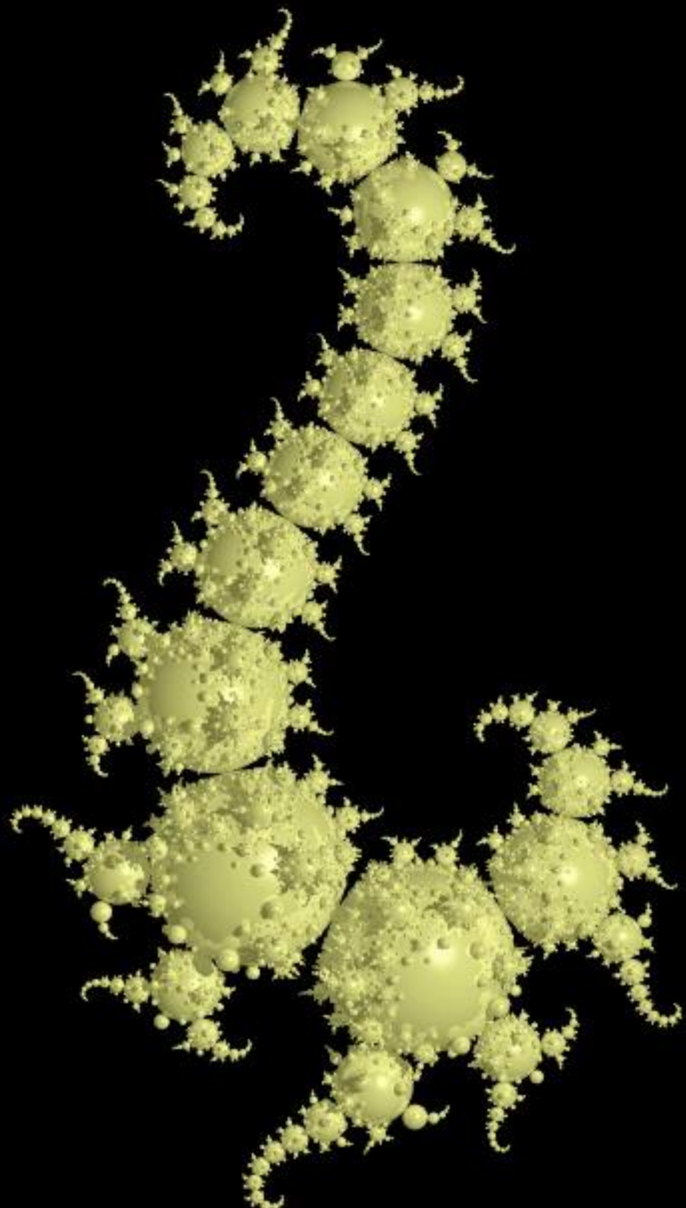


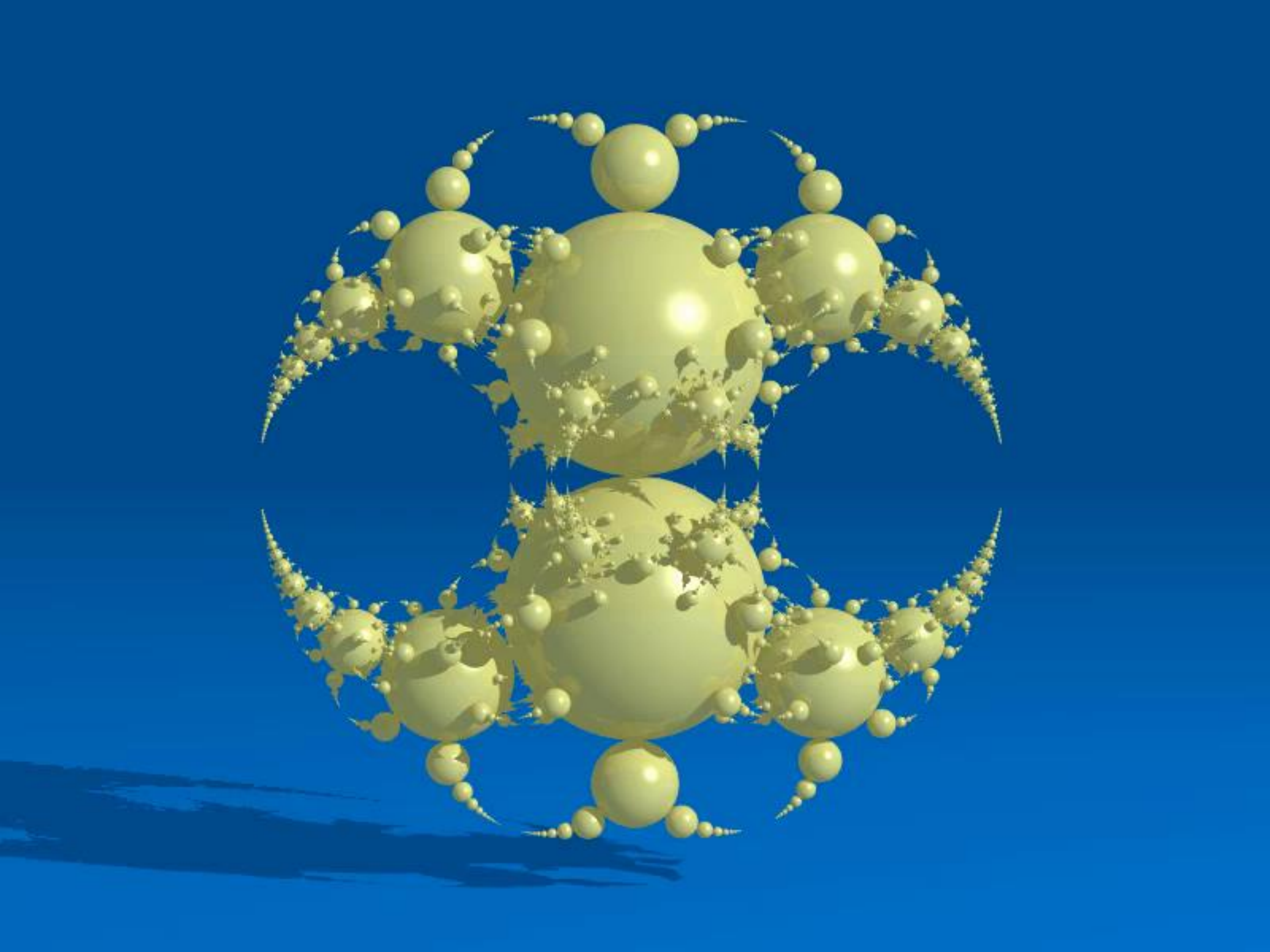


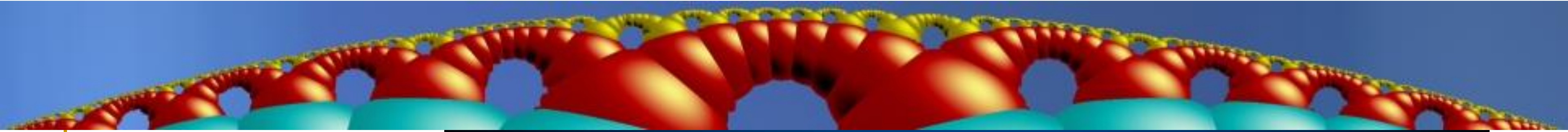




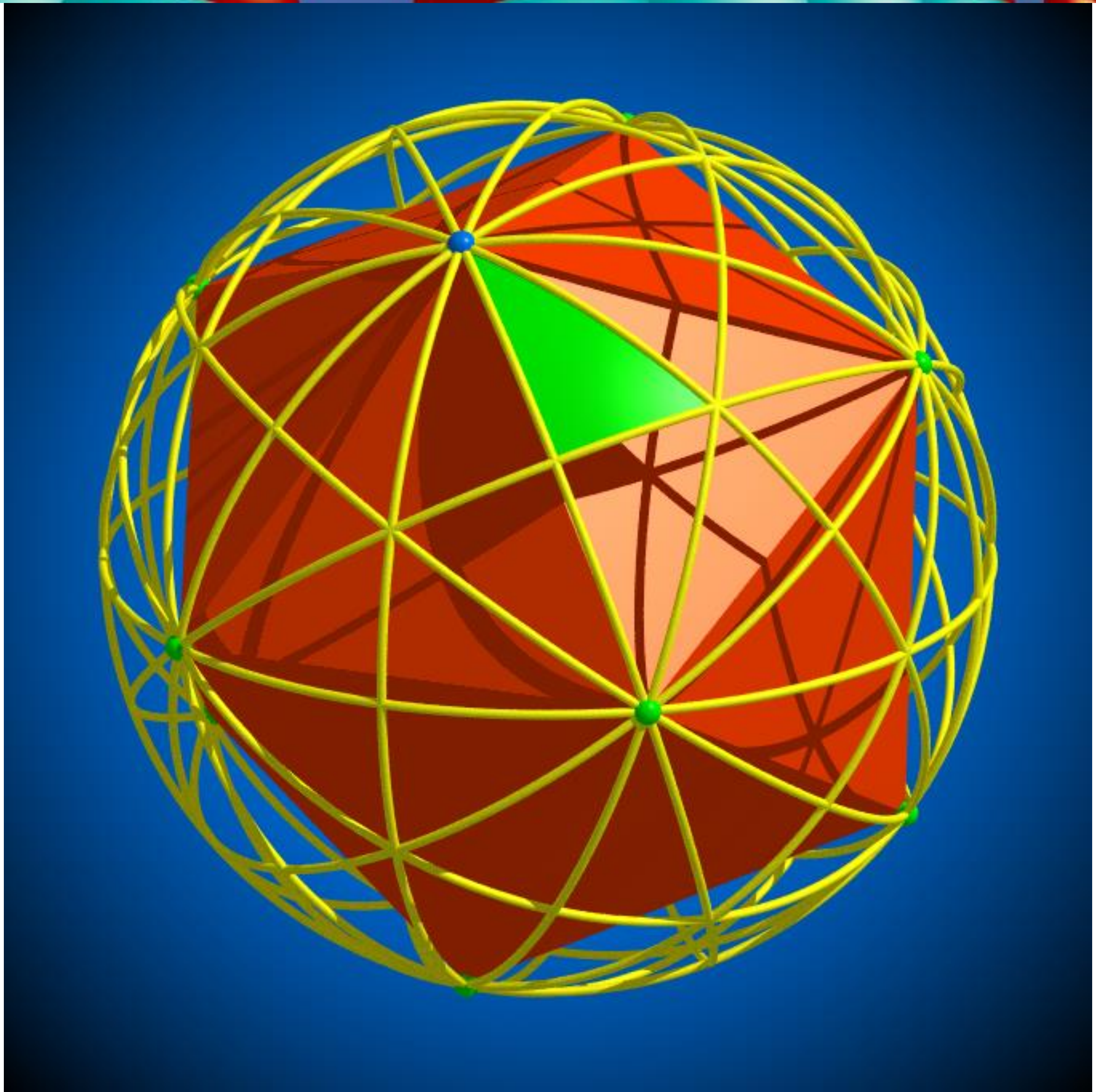




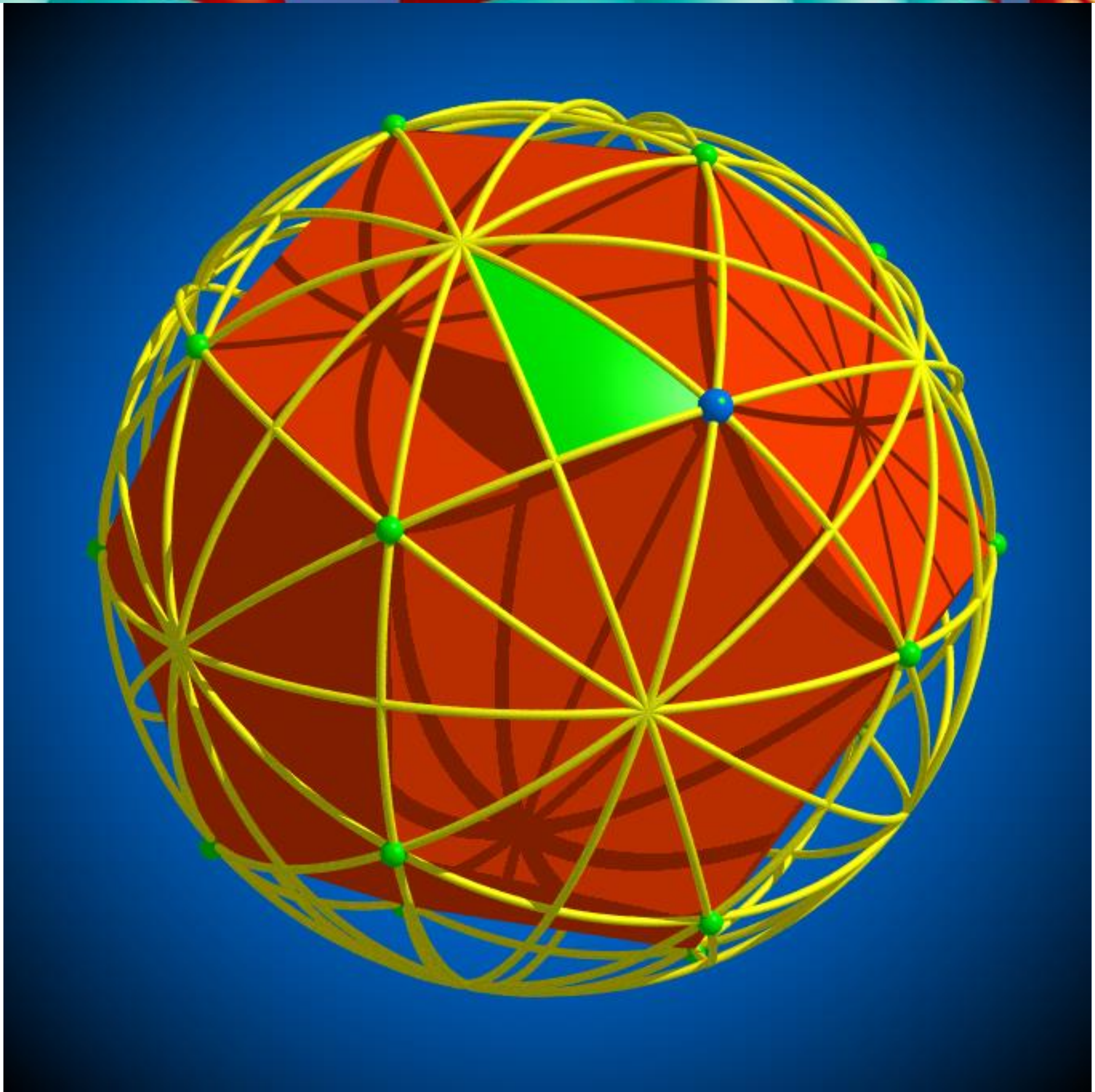


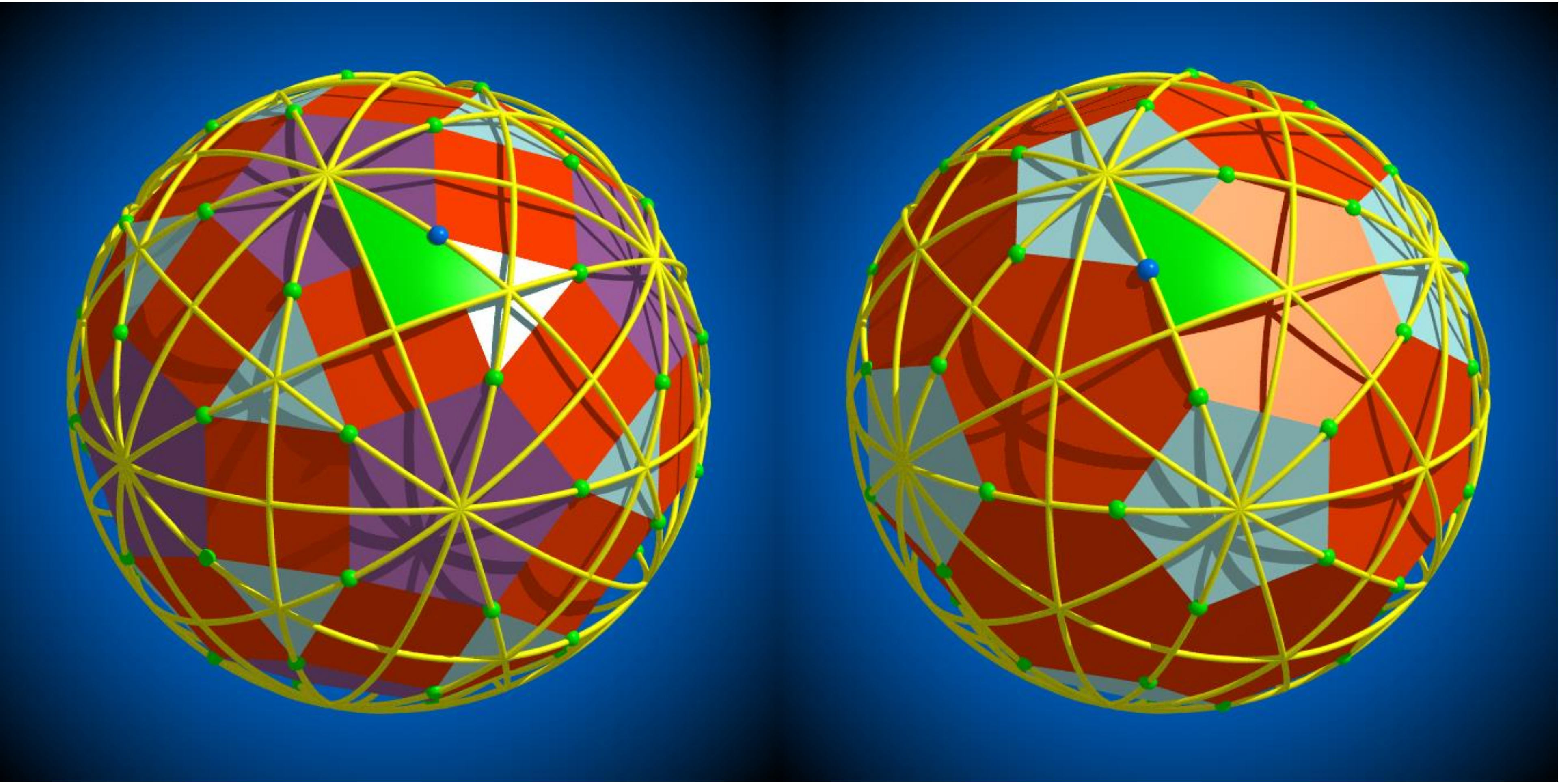


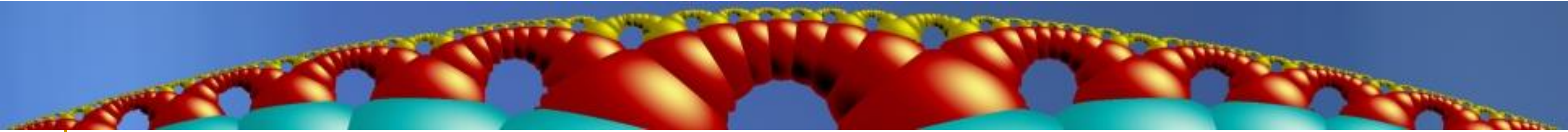
Kaleidoscopic method for polyhedra.



Kaleidoscopic method for polyhedra.

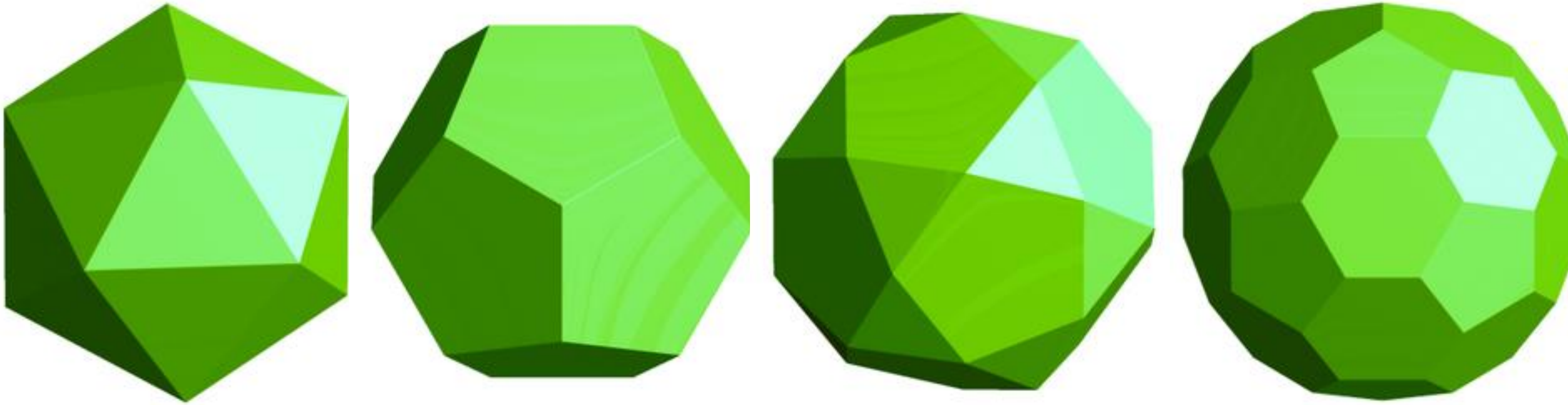




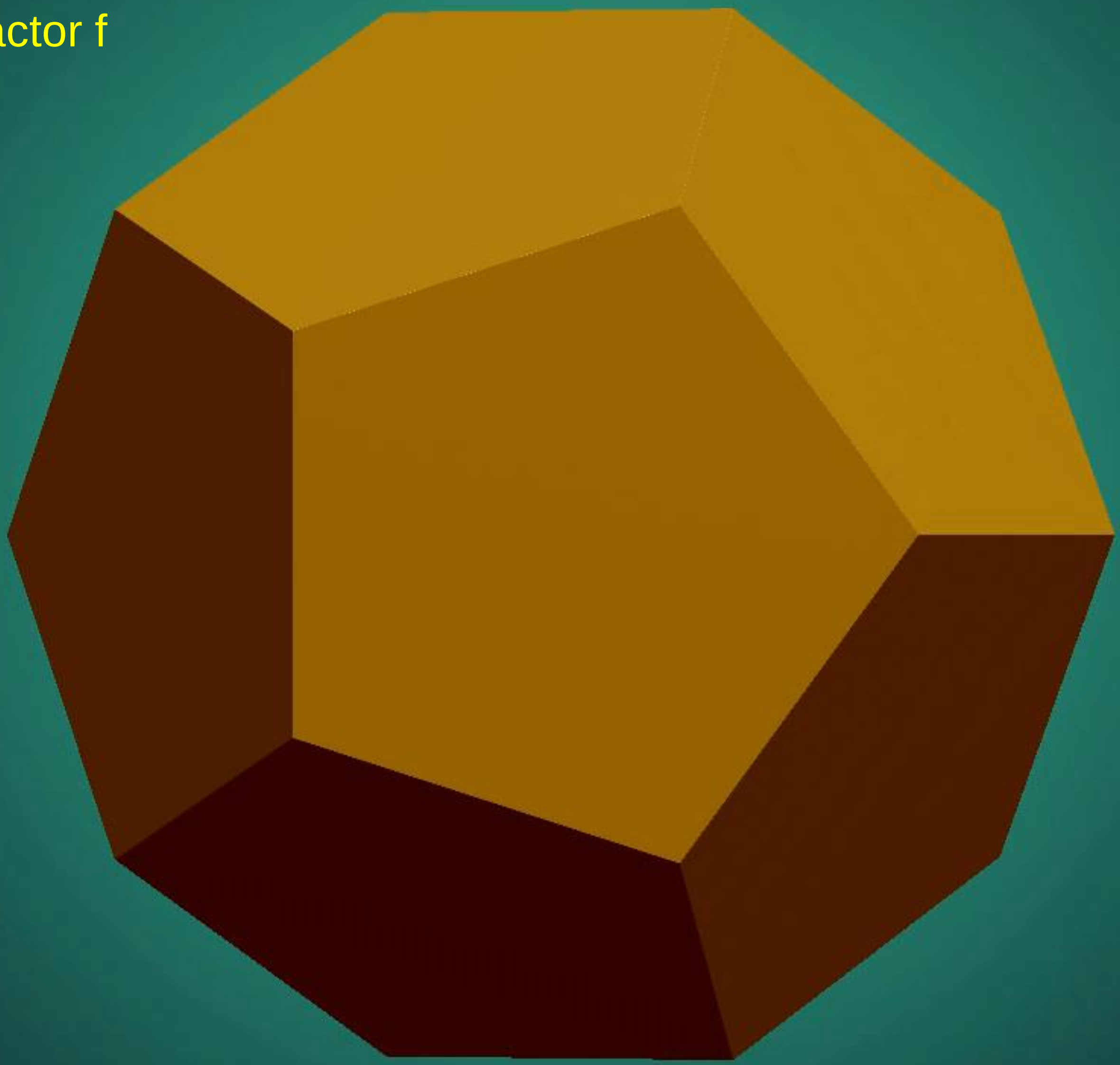


Algorithm

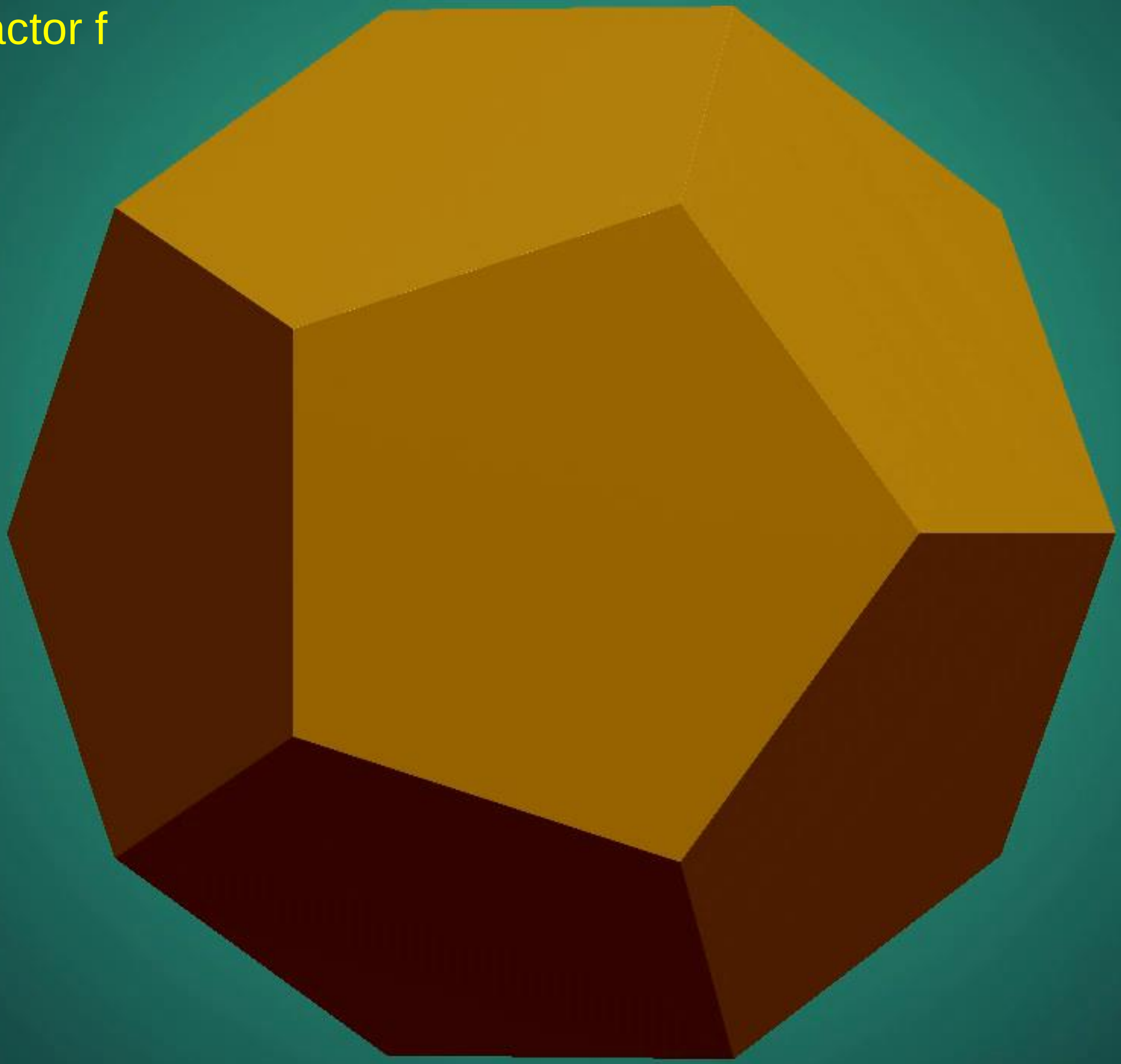
- ‘Fold’ point P so it lies in pyramidal area determined by triangle
- Transform $P' = C + f(P - C)$ ($f = 1.5$)
- If distance to origin<M and iterations<max, repeat, if not, stop.
- Move point according to distance formula



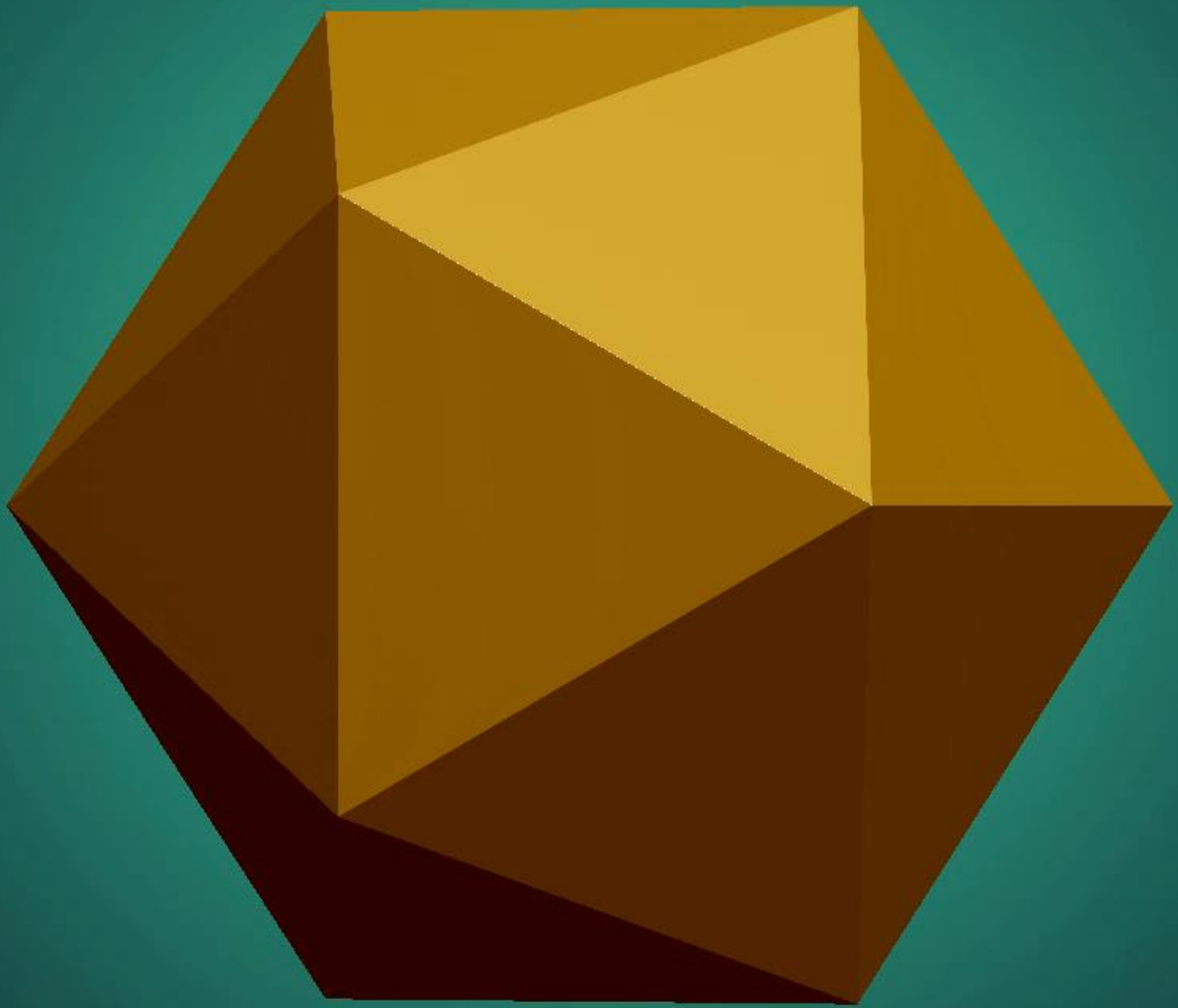
Increase factor f



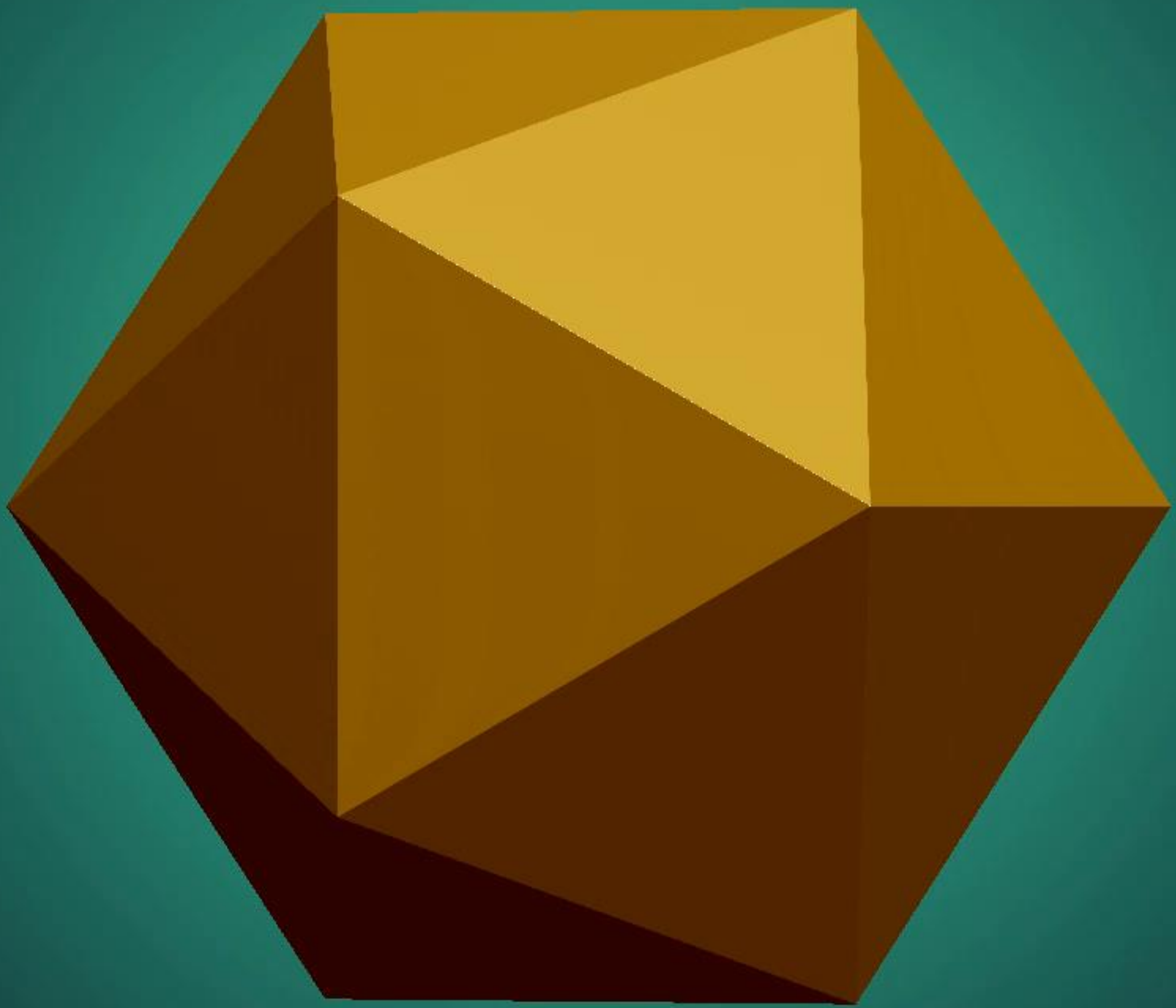
Increase factor f



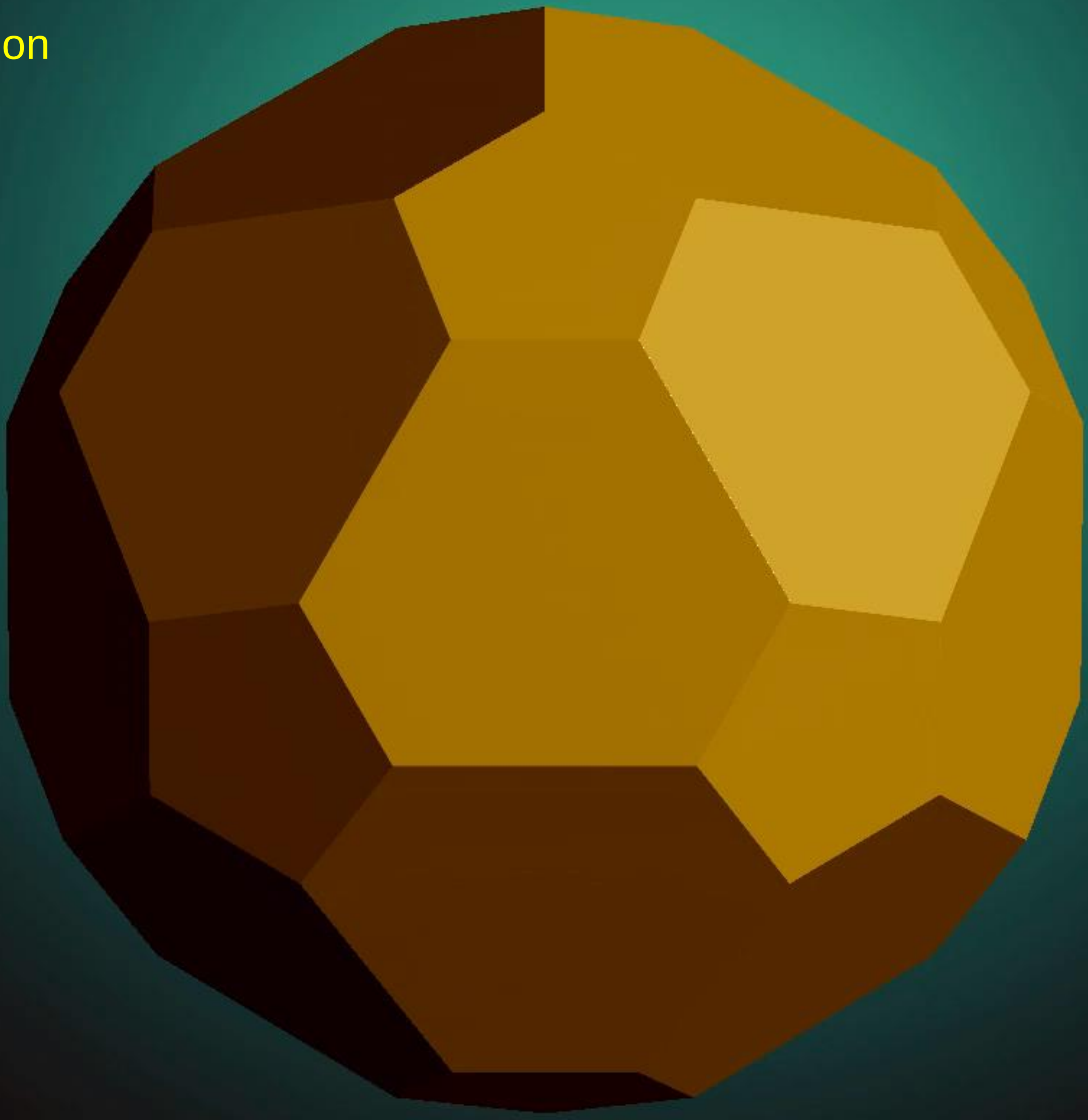
Increase factor f



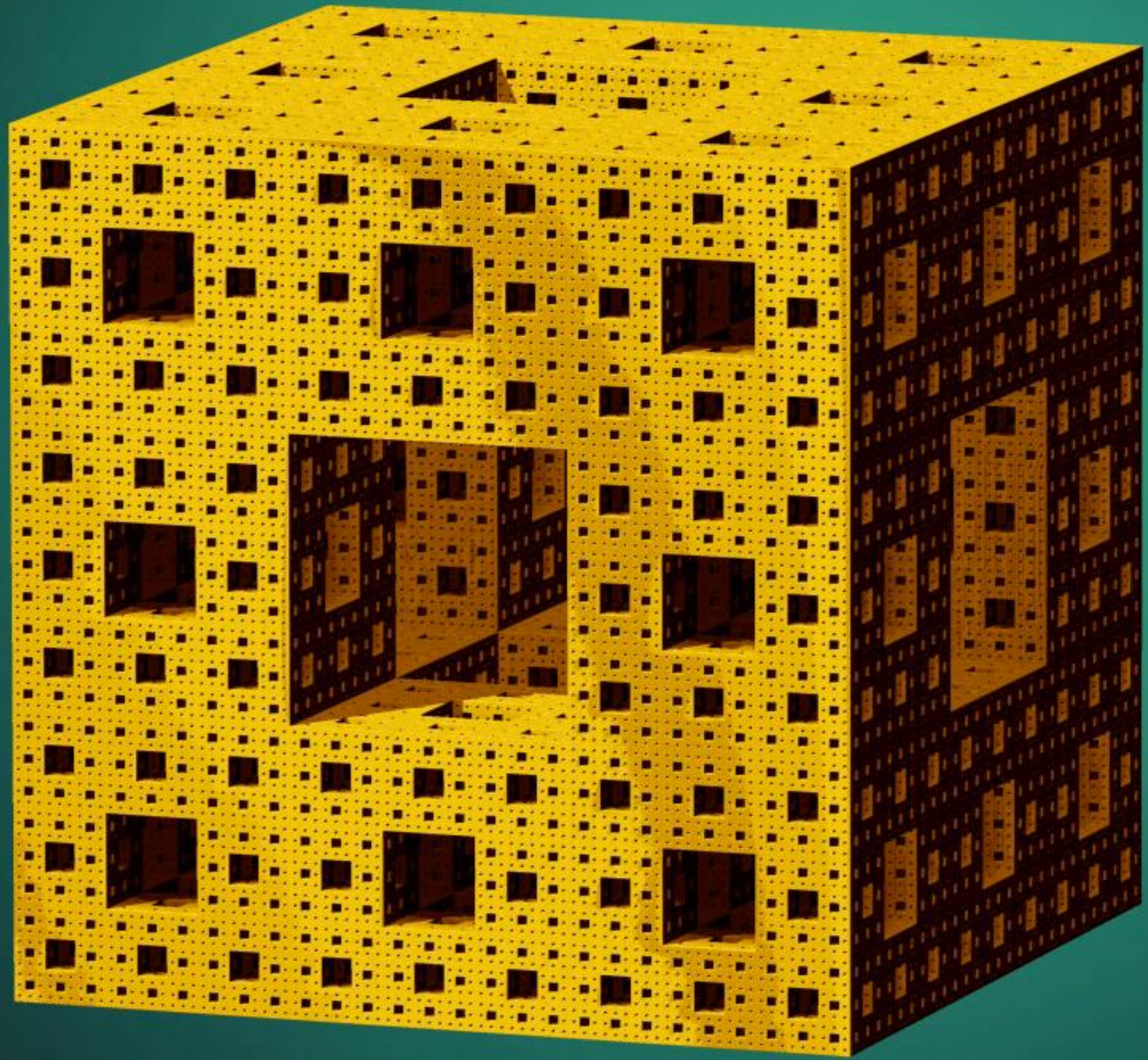
Increase factor f

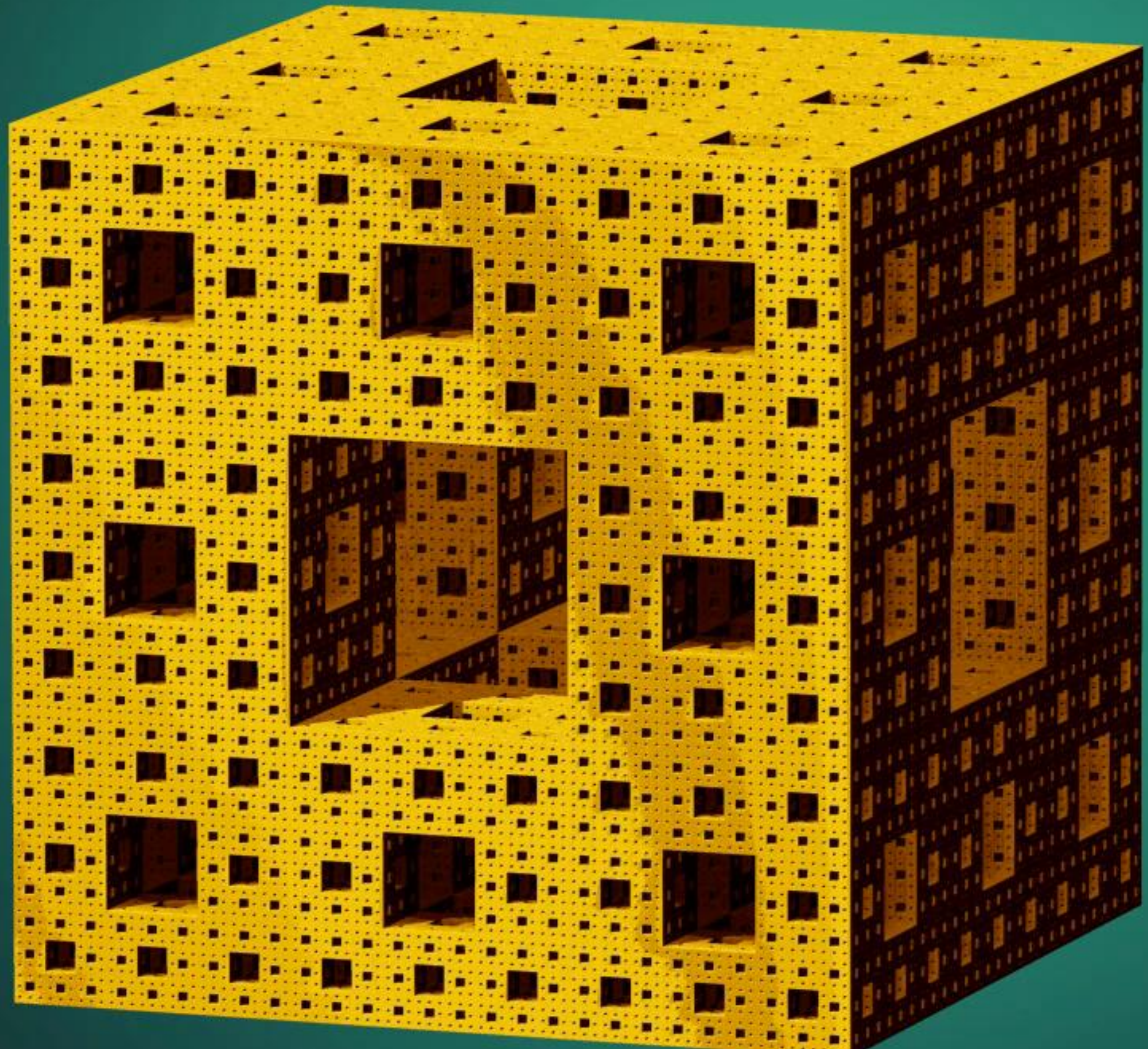


Add a rotation

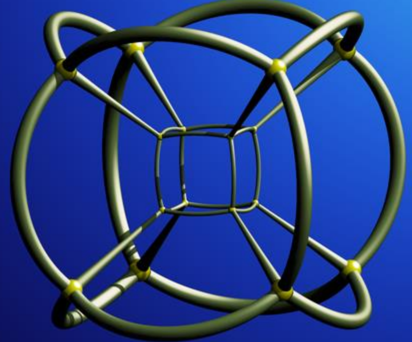


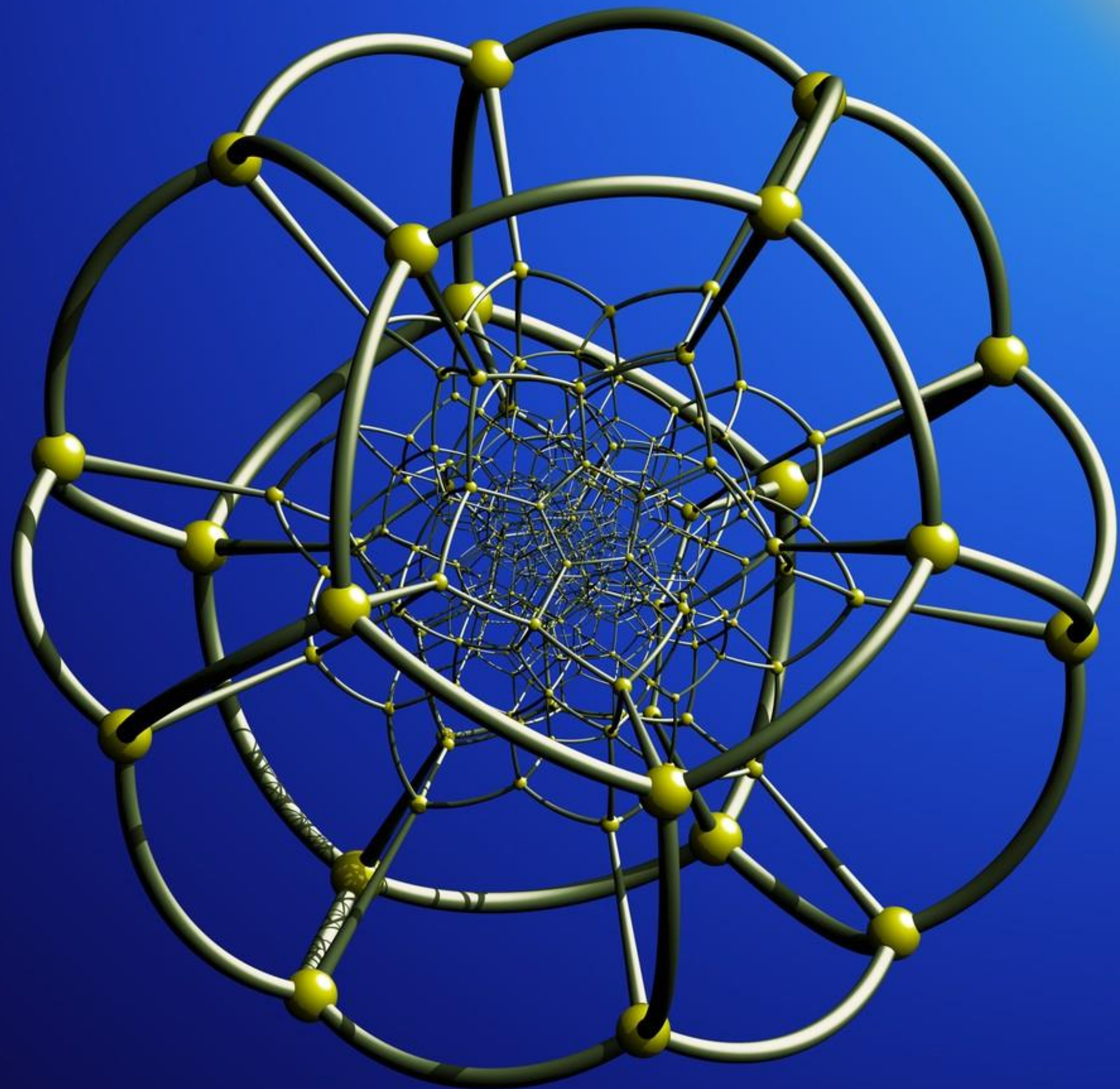
Add a rotation





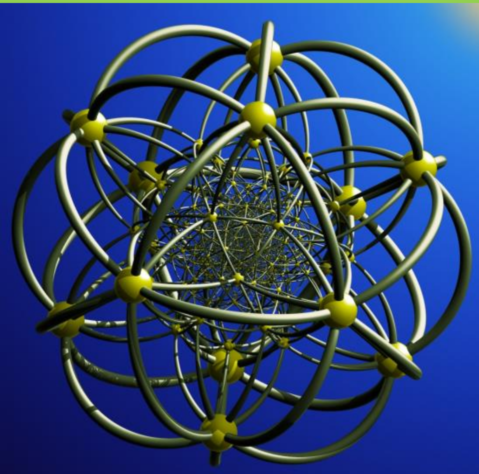
Kaleidoscopic method for 4D polychora.

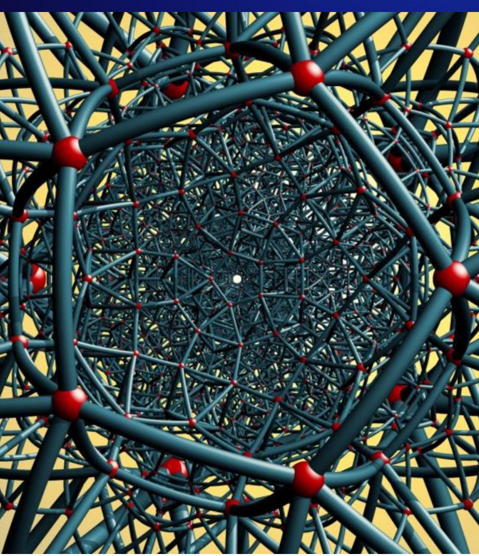


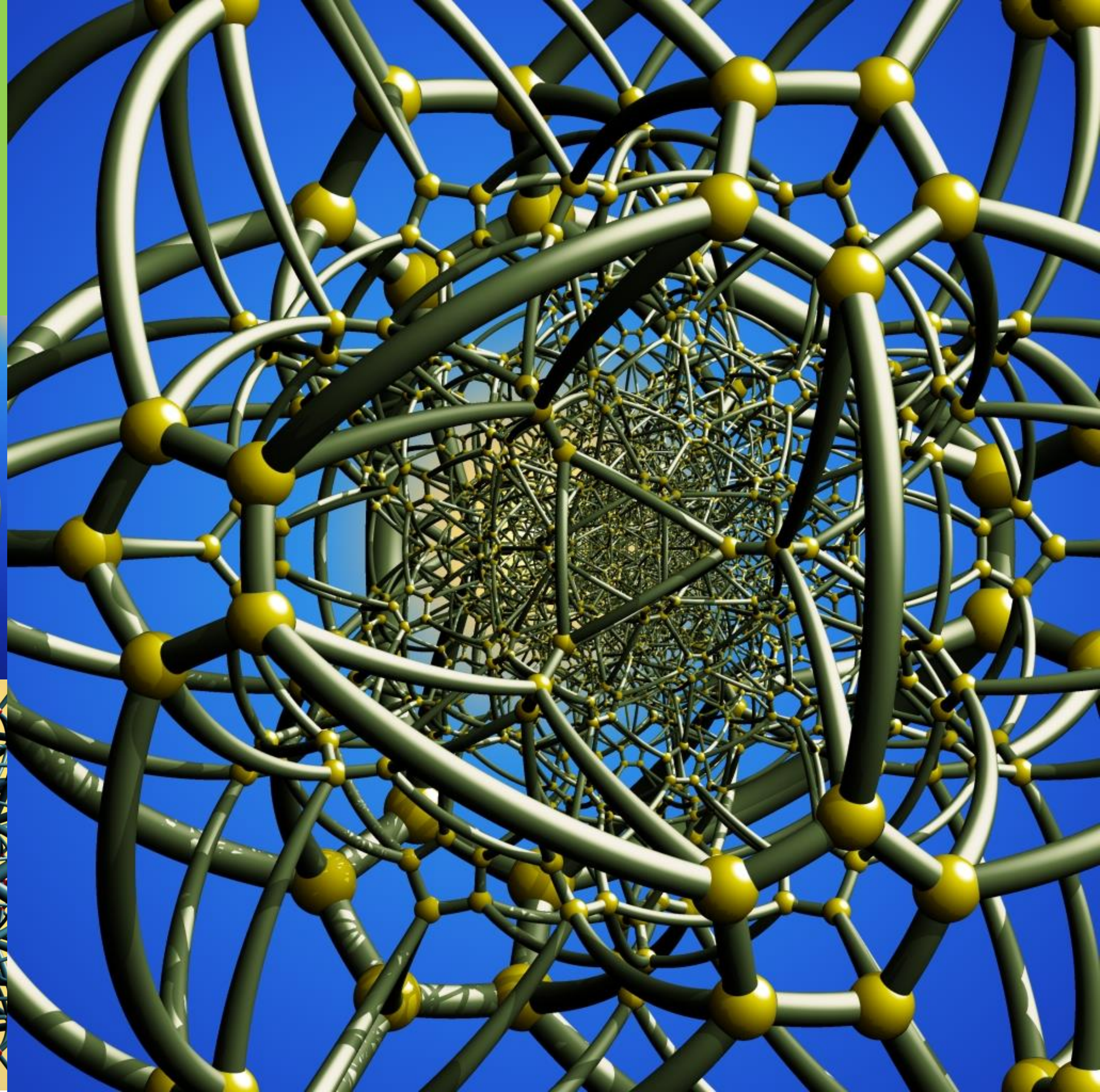


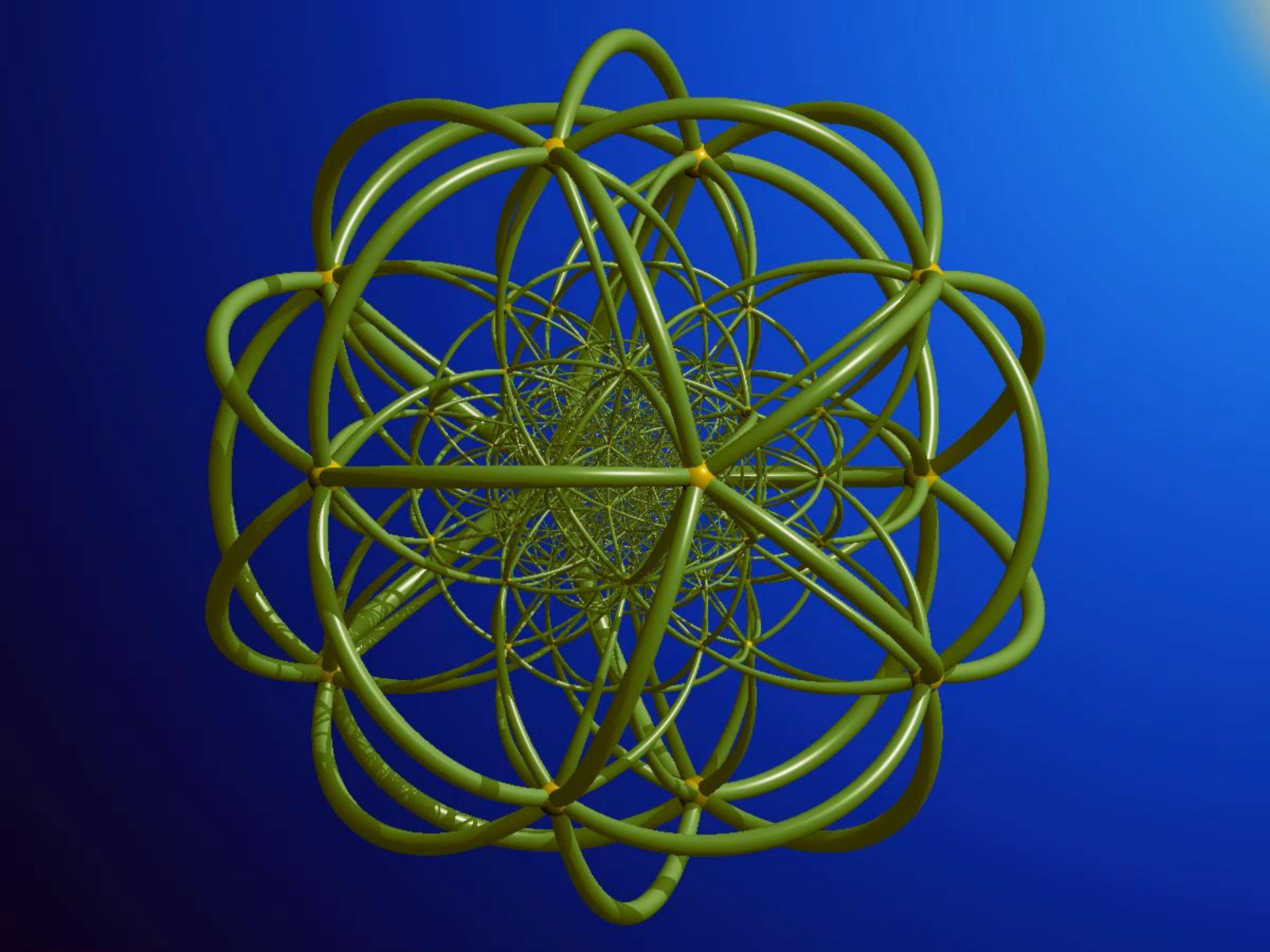
120-cell : 600 vertices, 1200 edges, 720 pentagons, 120 dodecahedra

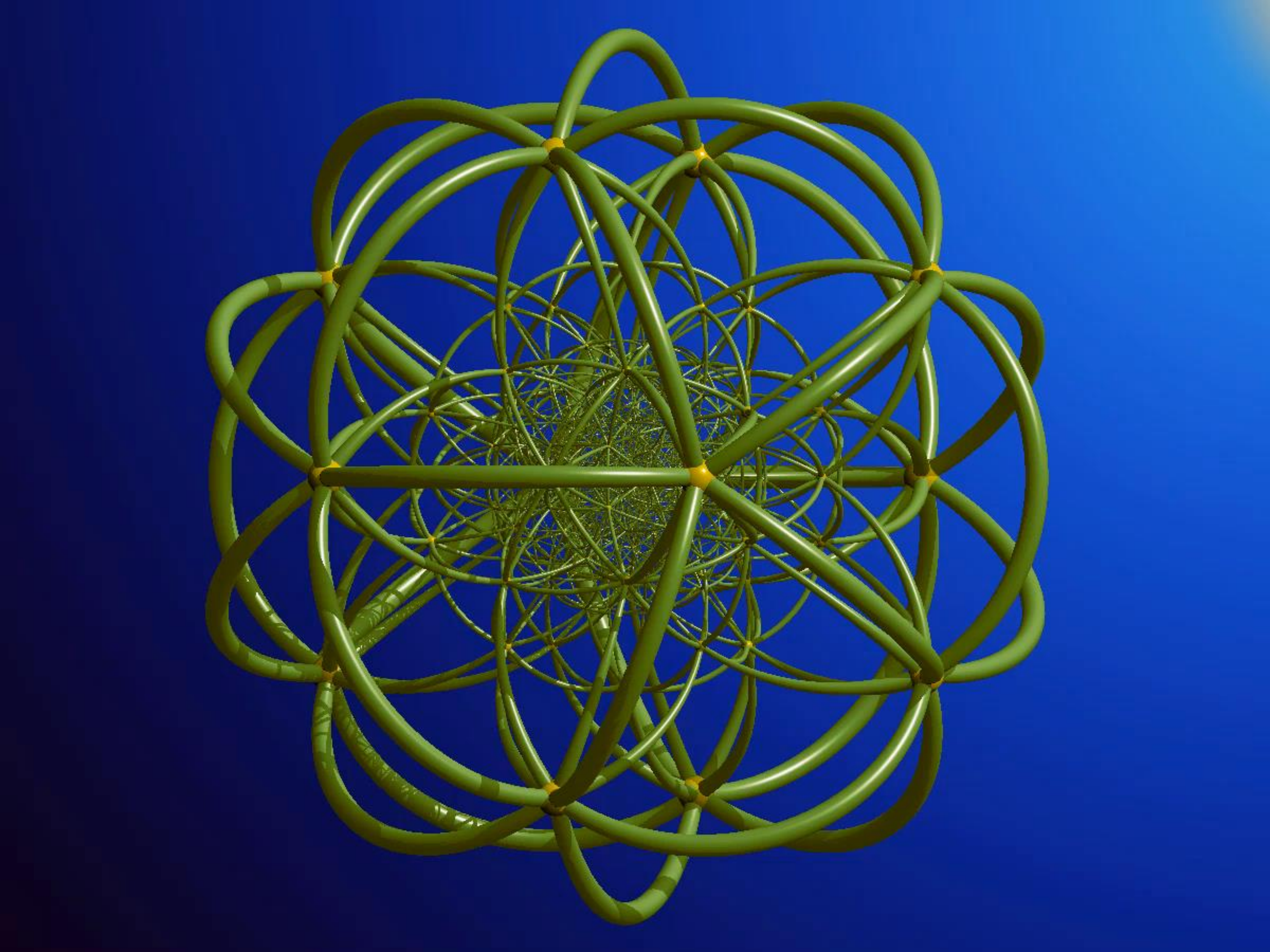
Kaleidoscopic method for 4D polychora.

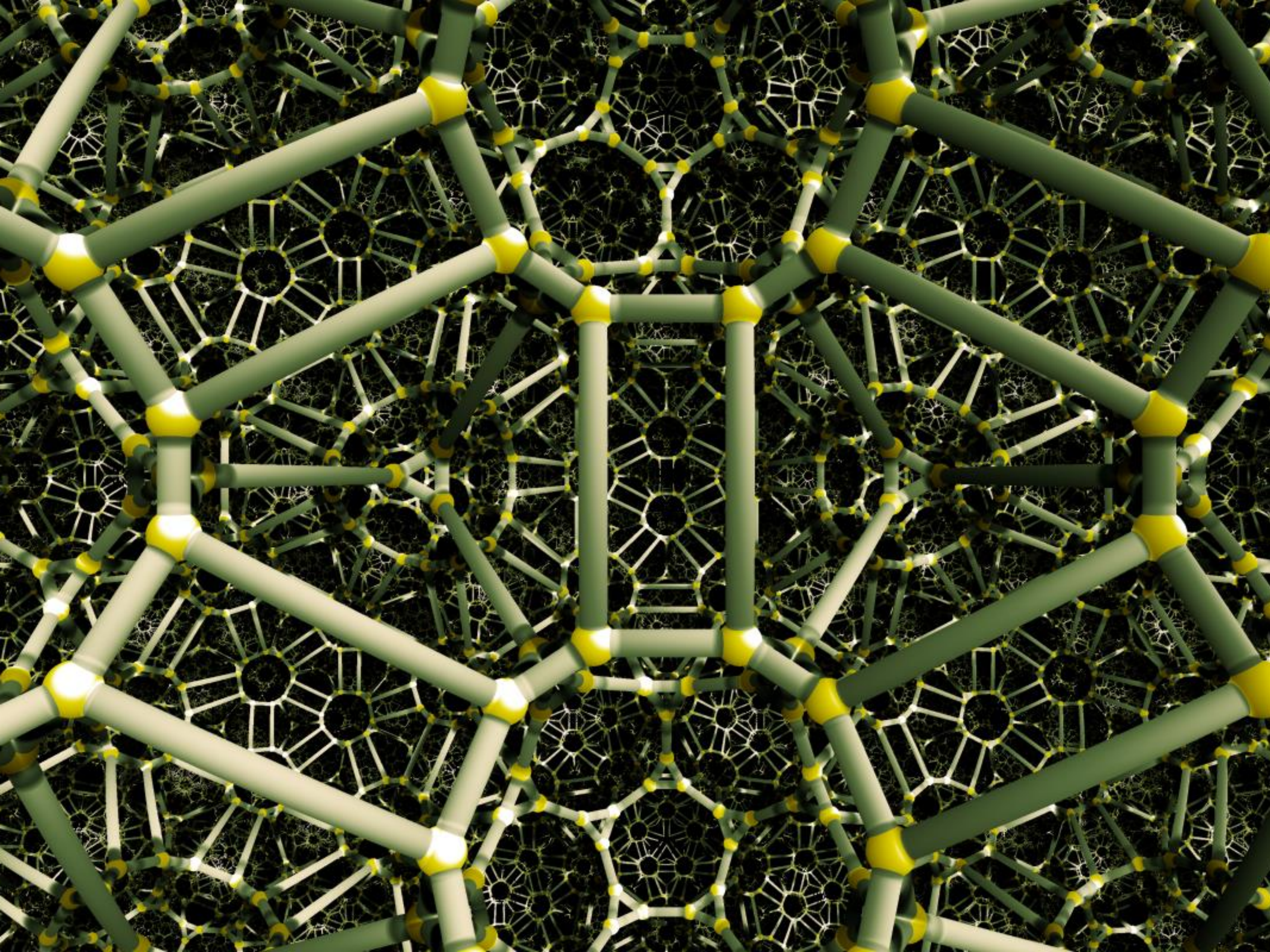


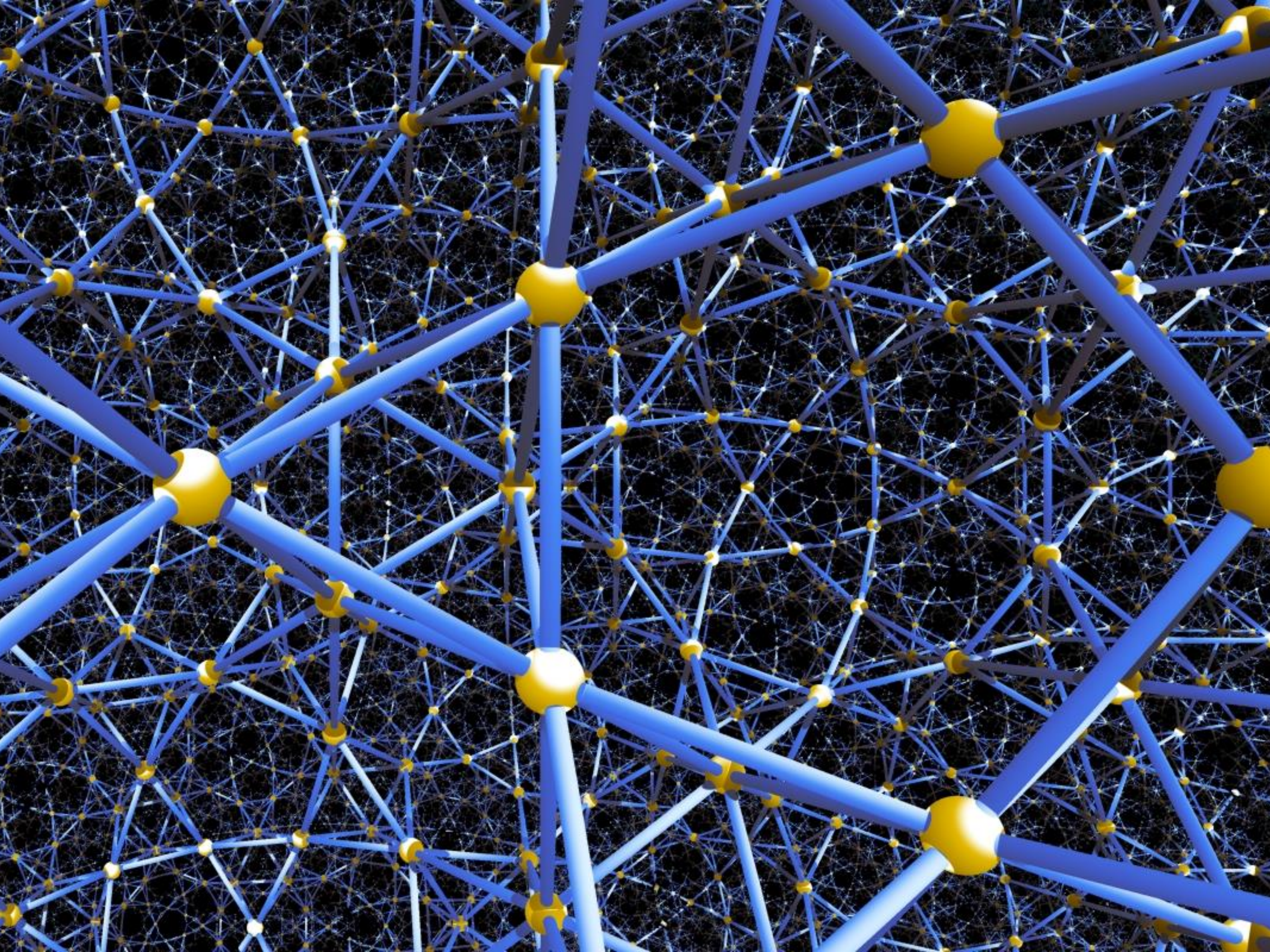






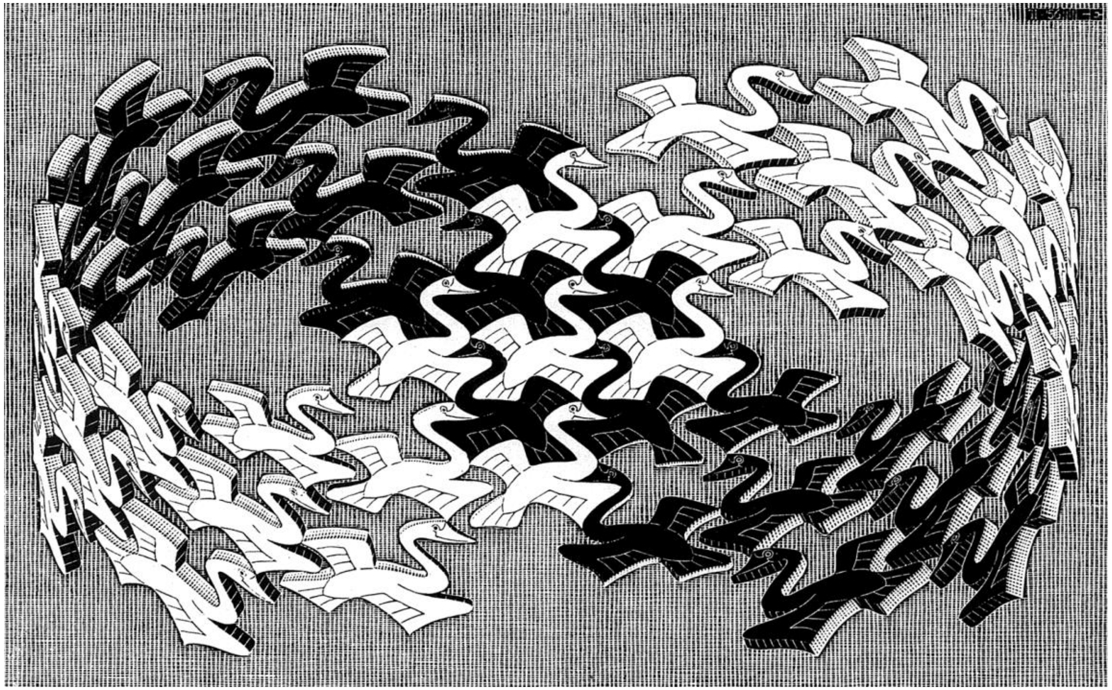


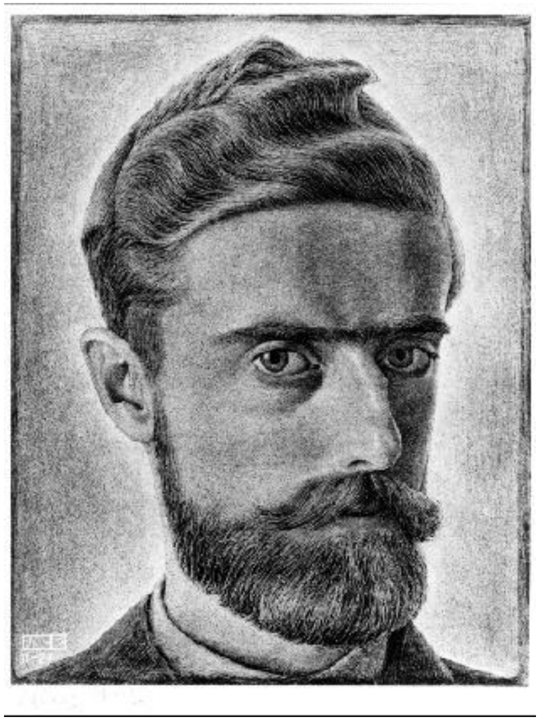


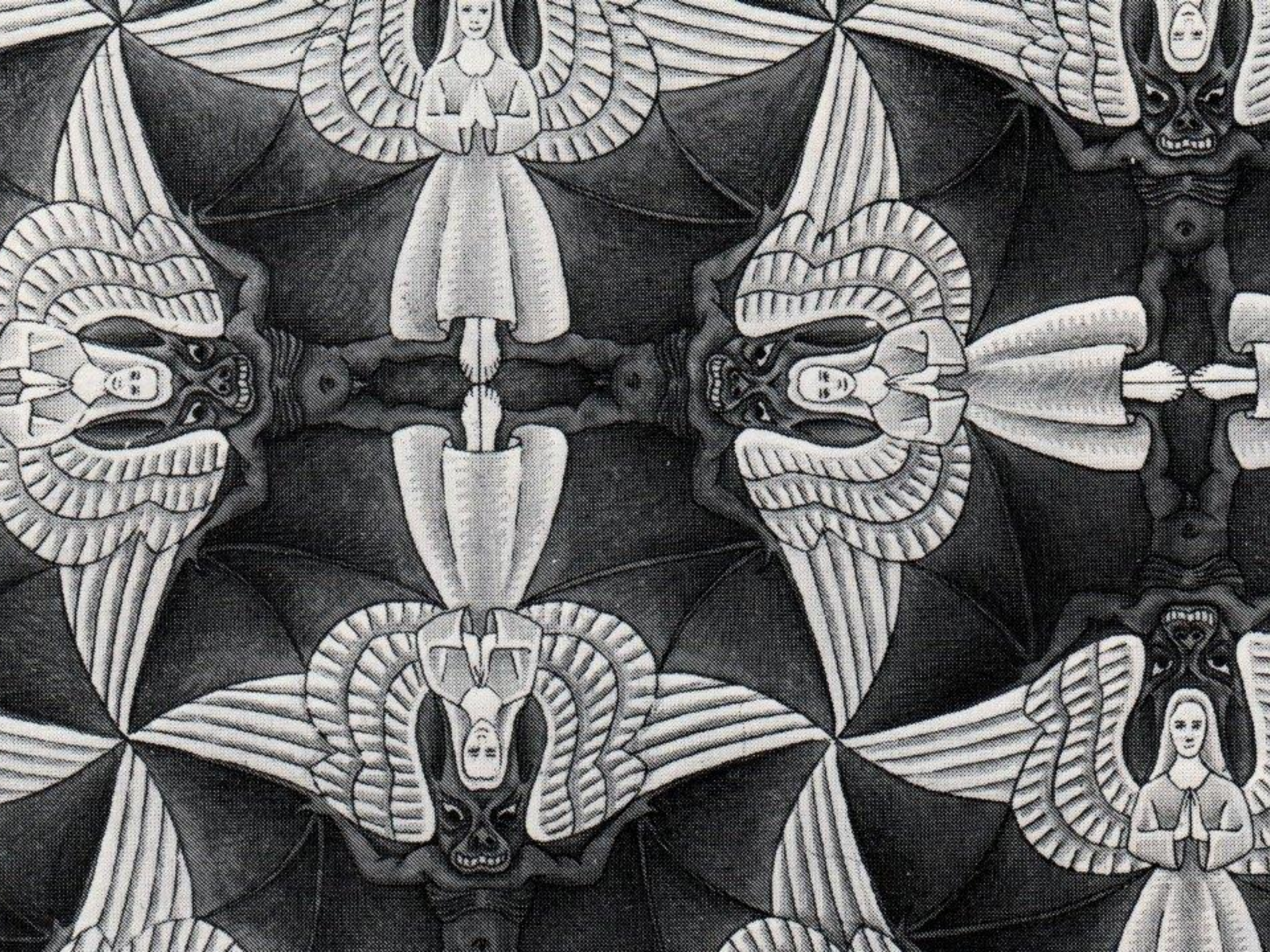


M.C. Escher

- Dutch artist (1898-1972)
- Studied graphic arts
- A lot of his work is inspired by maths








VACHERON CONSTANTIN
GENEVE

GENERAL | WHOLE INDEX CARD | PHOTOS

VACHERON CONSTANTIN | MORE ABOUT THE BRAND

Les Univers Infinis - Angel Watch - Vacheron Constantin

Brand : Vacheron Constantin
Collection : Métiers d'Art
Model : Les Univers Infinis

Reference : 86222/000G-9804
Nber of pieces : 20
Complement : Angel Watch

Sex : Men and Women
On sale : 2013

105 300 €
Recorded list price in France
I WANT IT

ADD THE WATCH TO MY DREAM BOX
SEND TO A FRIEND
GRADE THE WATCH (99 votes)
Like 2 Tweet
POINT OF SELL

PDF INDEX CARD

DESCRIPTION

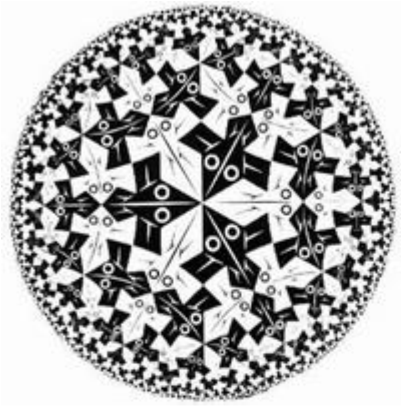
The Artistic Crafts of the Manufacture pay tribute to the art of tessellation

• Second series in the Métiers d'Art Les Univers Infinis collection, inspired by the work of the Dutch artist Maurits Cornelis Escher
• A tribute to the graphic expression of the periodic paving technique known as [...]

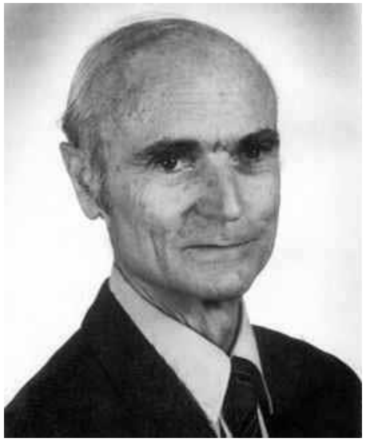
SEE THE WHOLE DATA SHEET

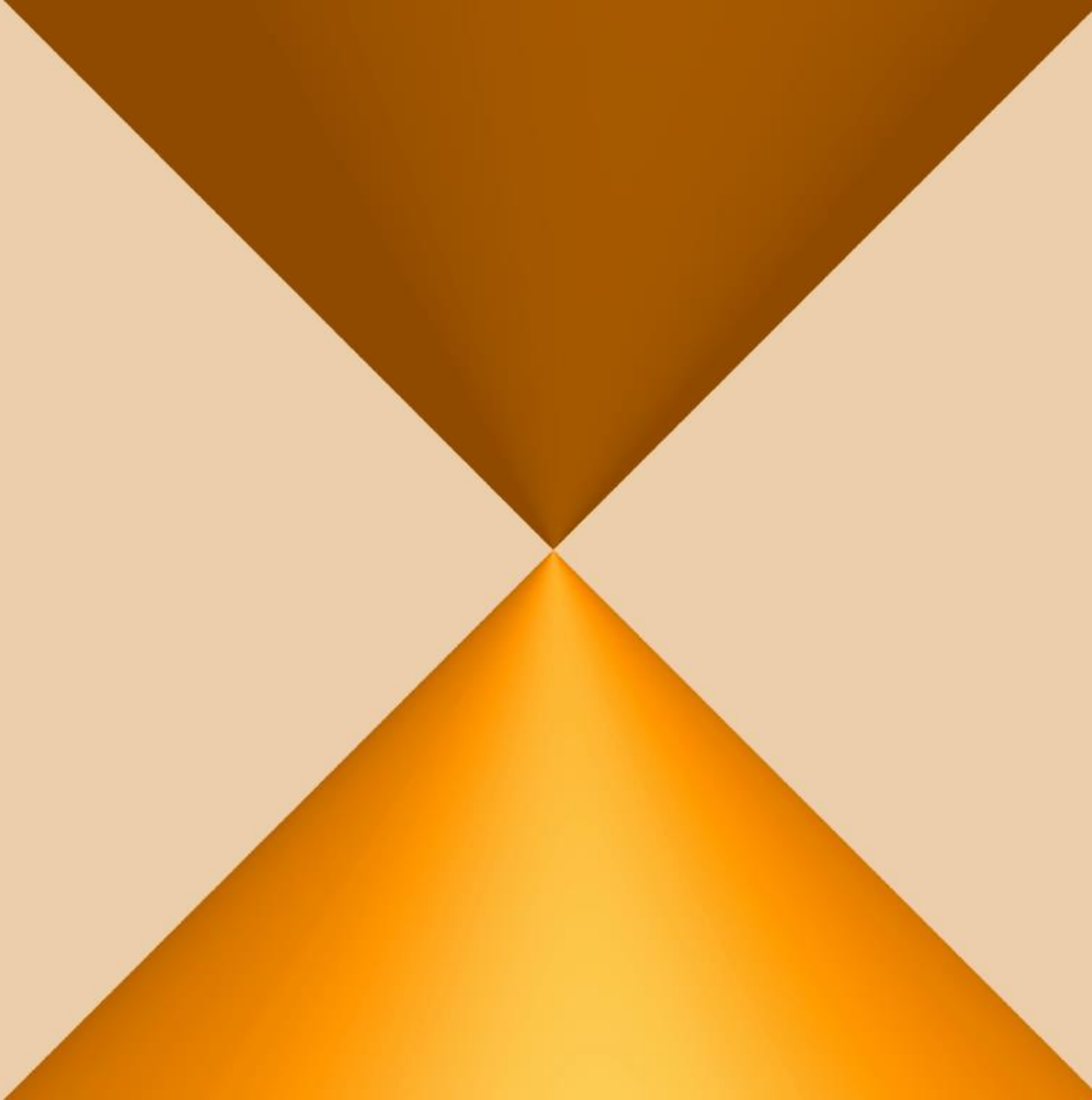
Hyperbolic Escher

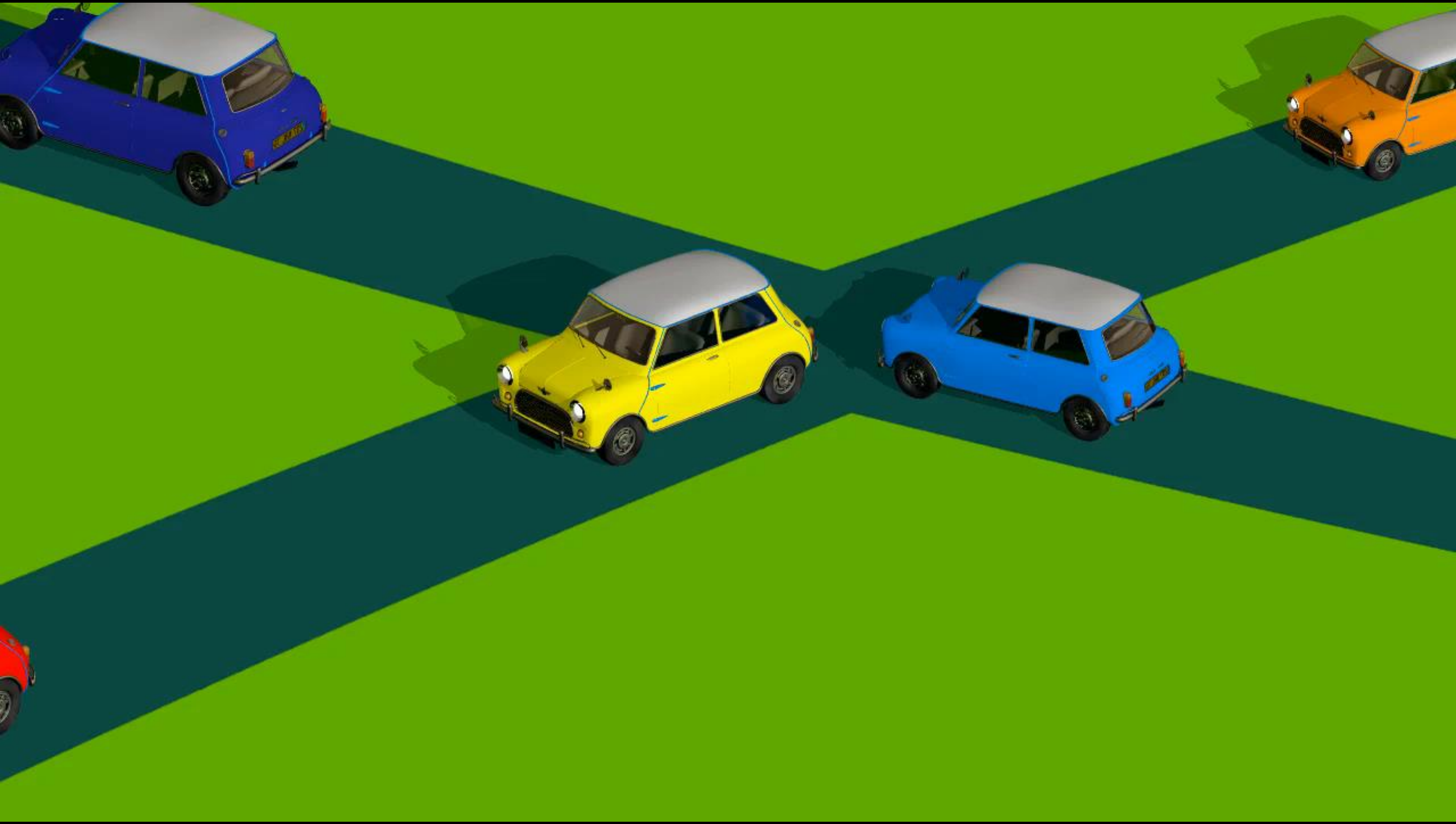
- Escher created 4 hyperbolic tilings



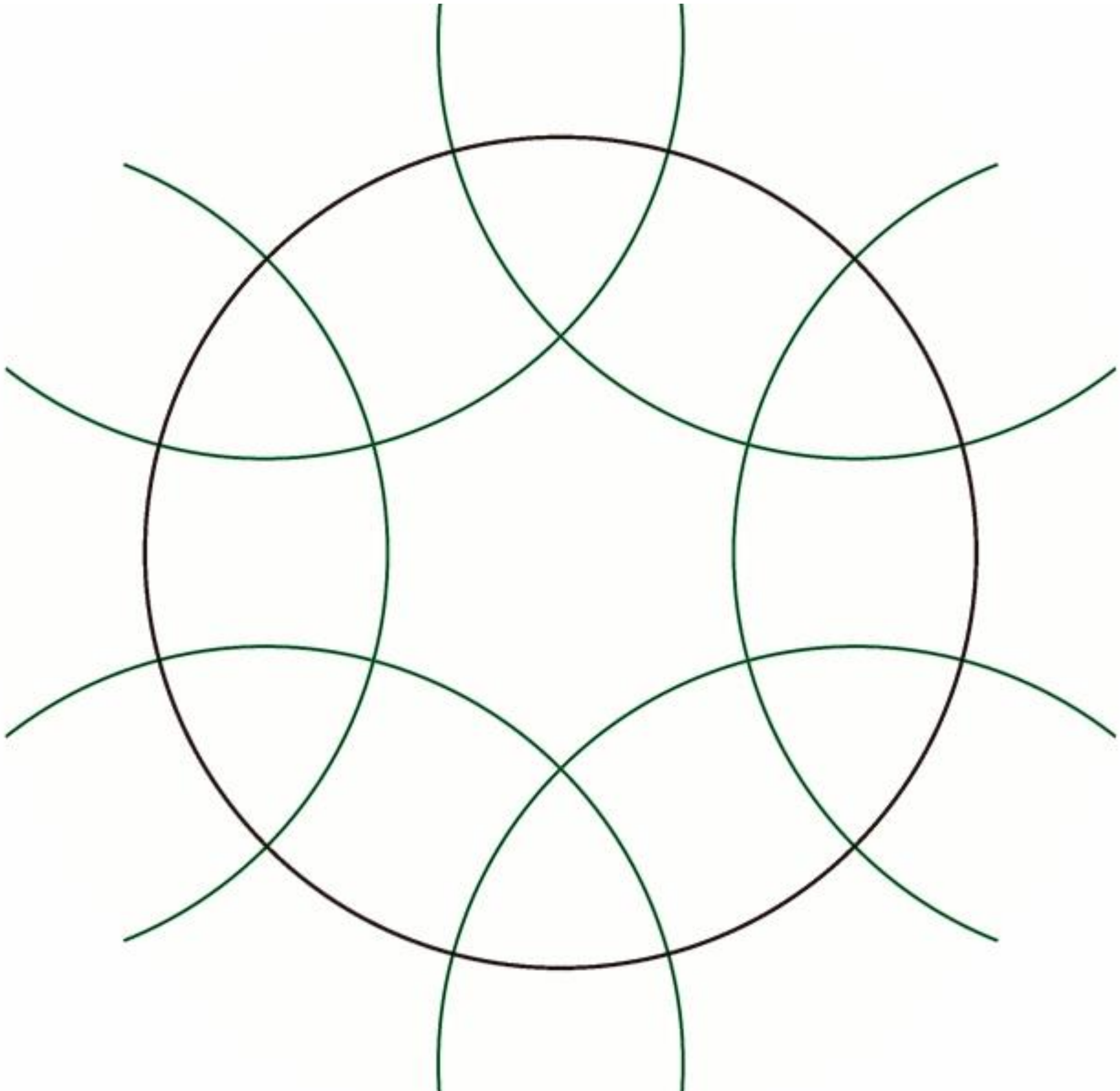

- He got help from mathematician H.M. Coxeter



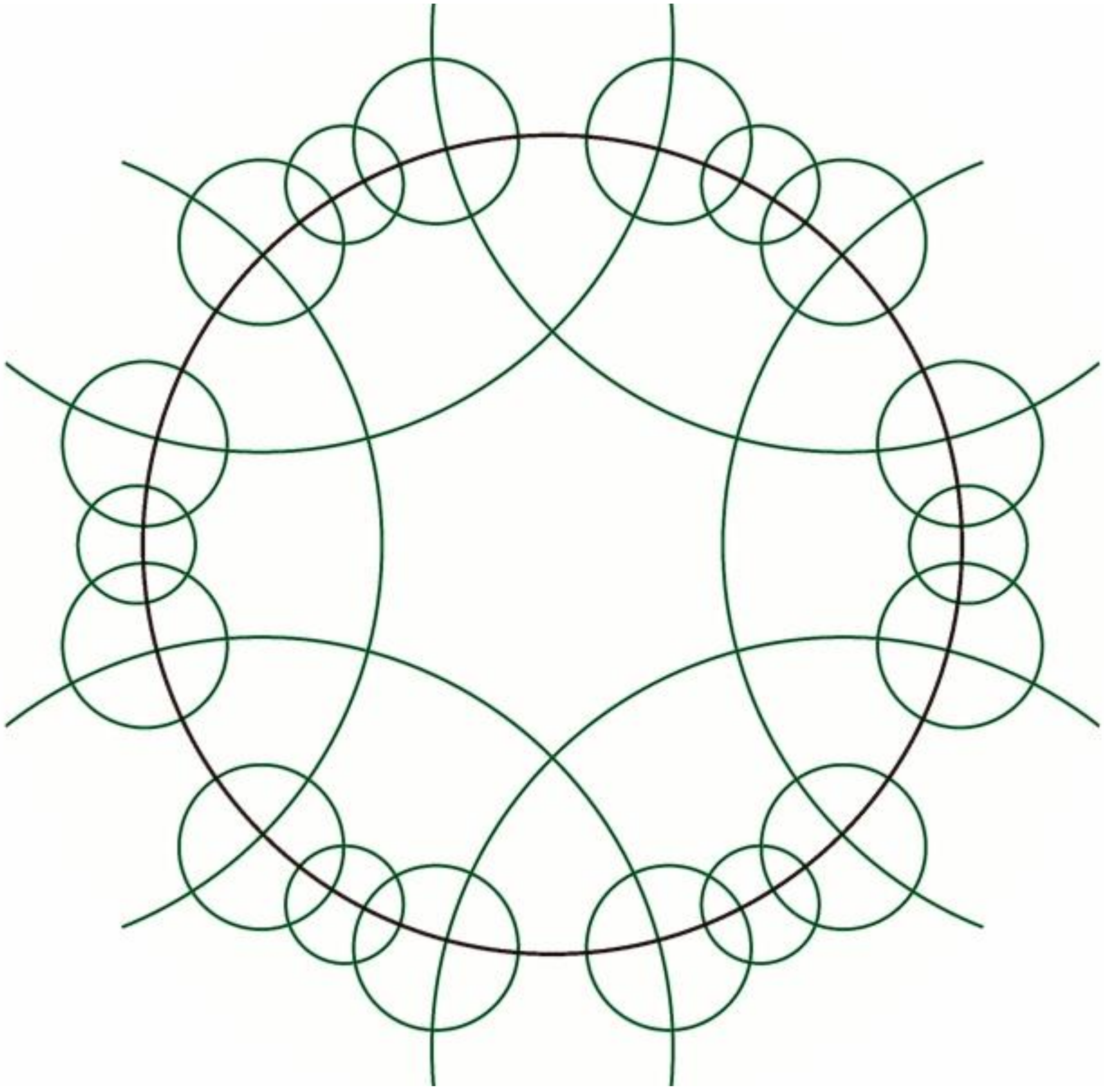




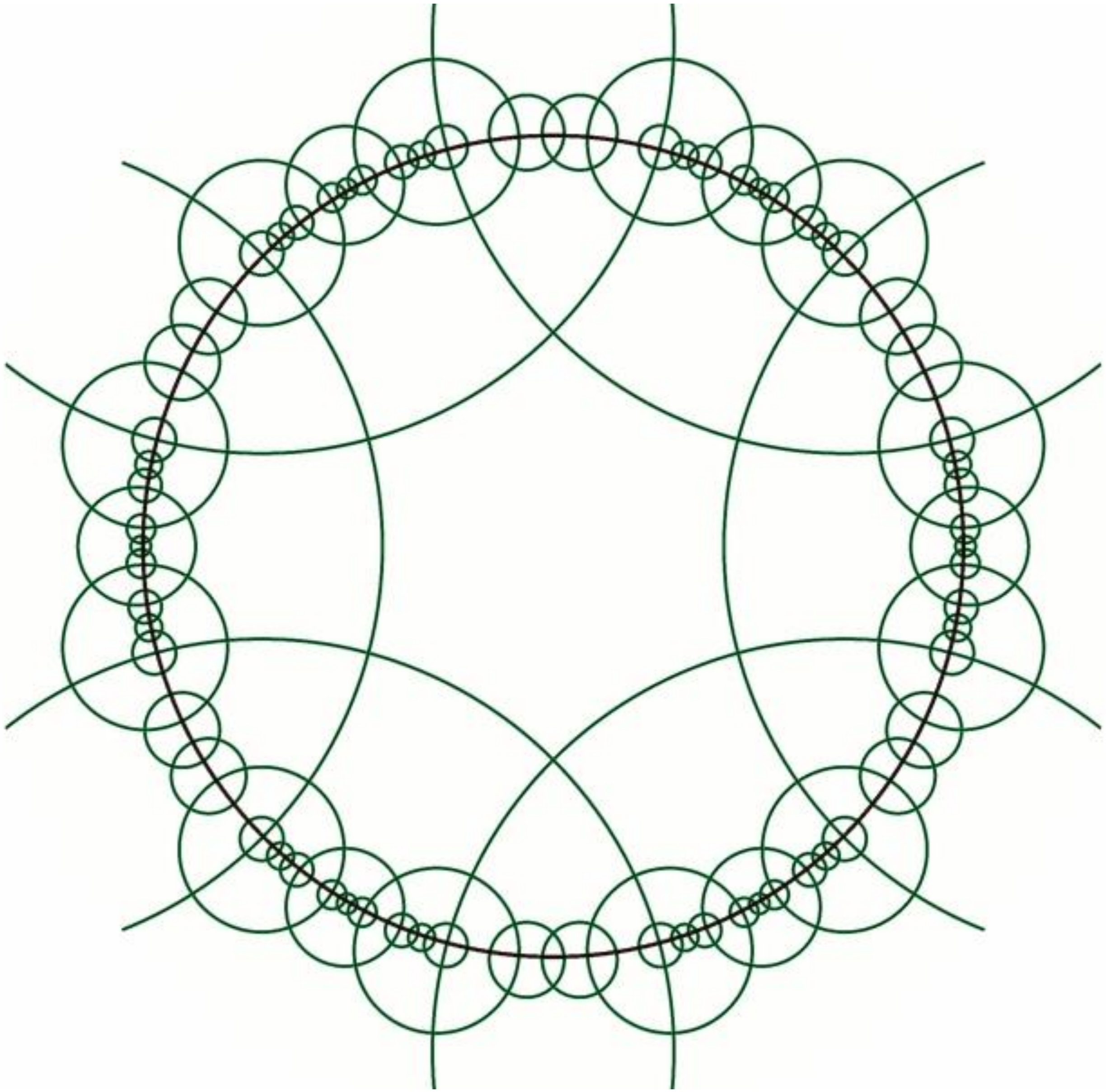
7 circles

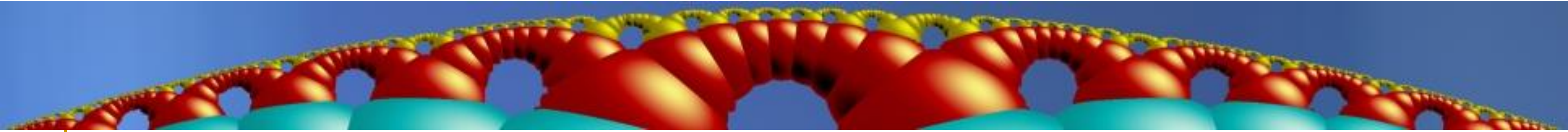


25 circles

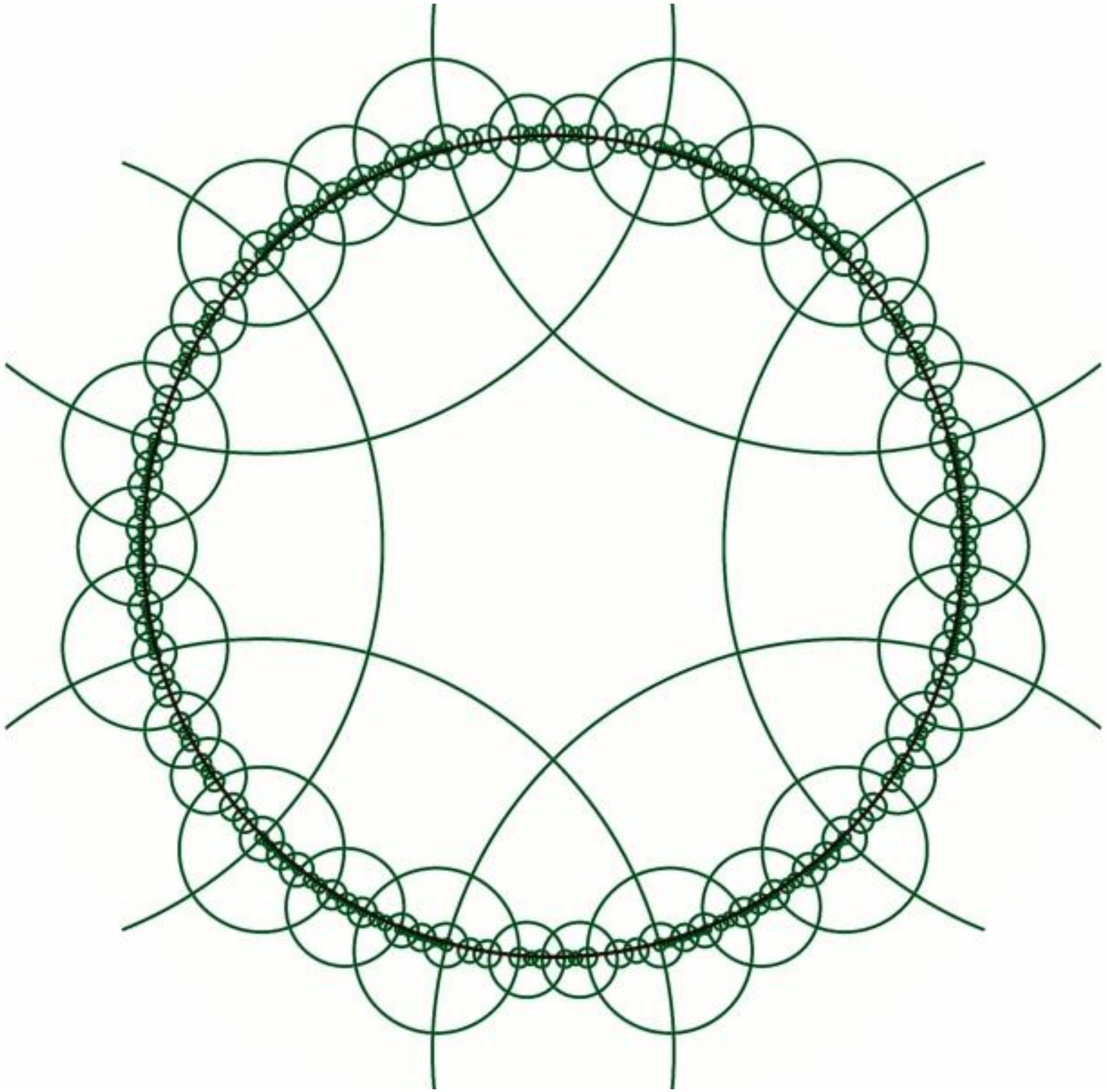


103 circles

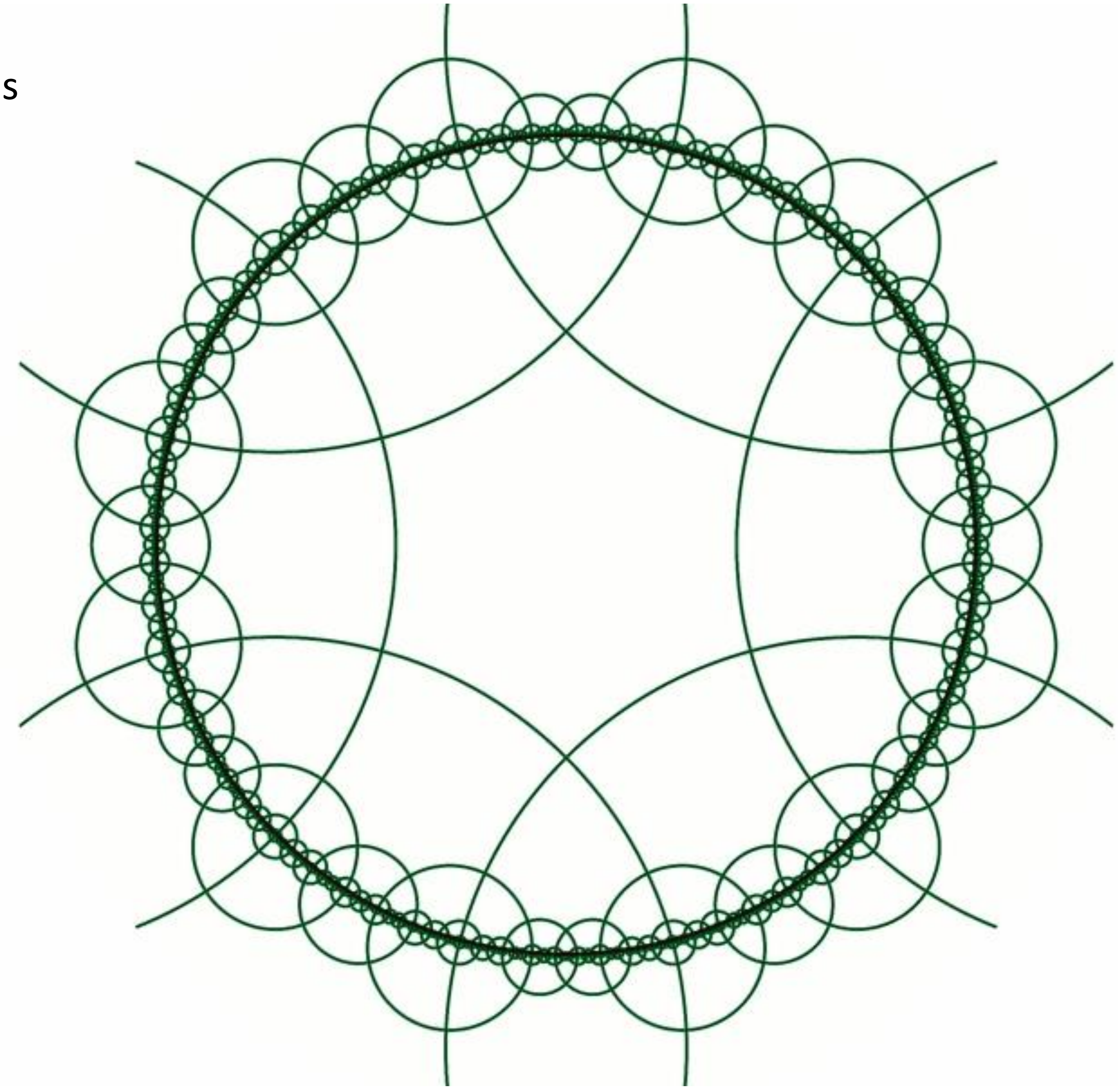


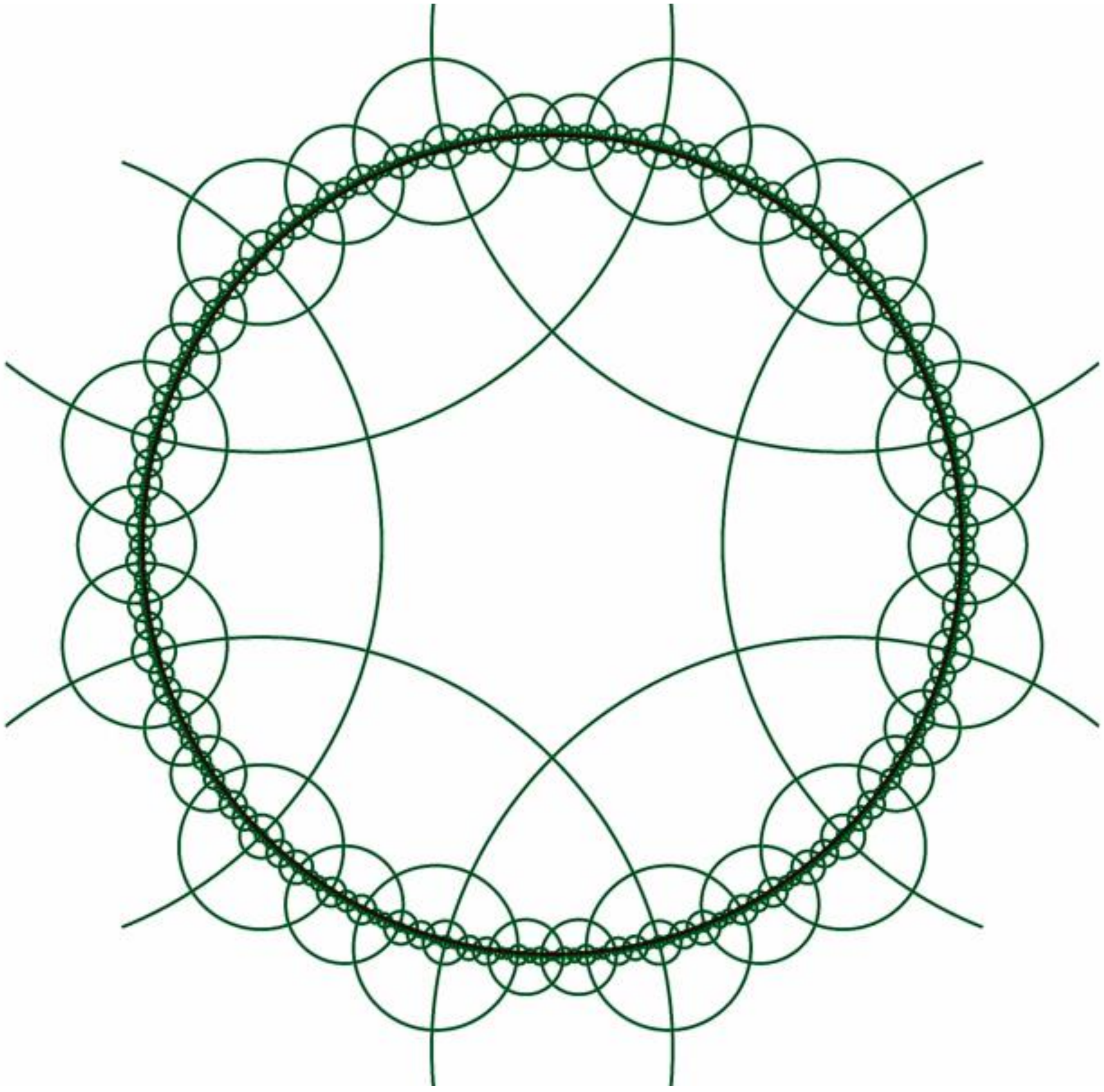


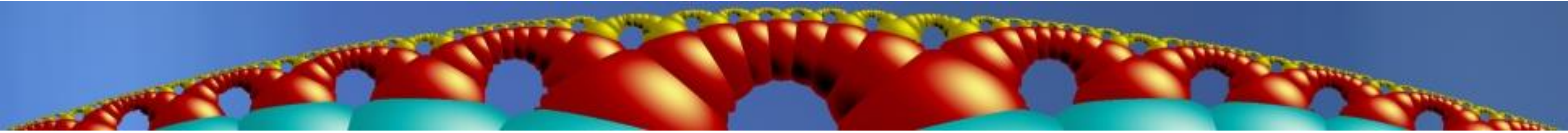
403 circles



>1300 circles







M.C. Escher, *CIRCLE LIMIT IV (HEAVEN AND HELL) (B./K./L./W. 436)* — Auction Result

TITLE	*CIRCLE LIMIT IV (HEAVEN AND HELL) (B./K./L./W. 436)*
CREATION DATE	1960
MEDIUM	Woodcut printed in black and ochre
SIZE	16 3/8 in 41.6 cm

AUCTION HOUSE	Sotheby's New York
AUCTION DATE	Oct 31 - Nov 1, 2013
ESTIMATE	$20,000 - 30,000
SALE	$46,875

Visit the lot on sothebys.com

(NOTE: Auction houses occasionally remove older lots from their websites)

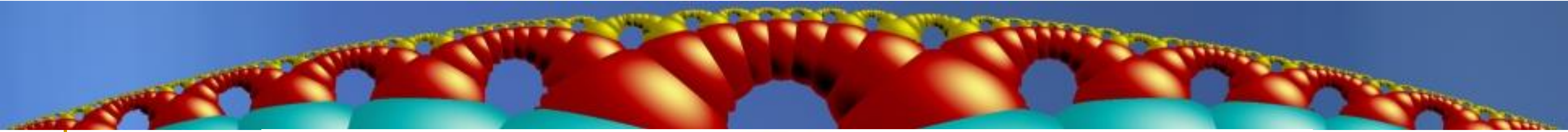
M.C.Escher
Ringslangen, 1969

M.C. Escher, *Snakes* — Auction Result

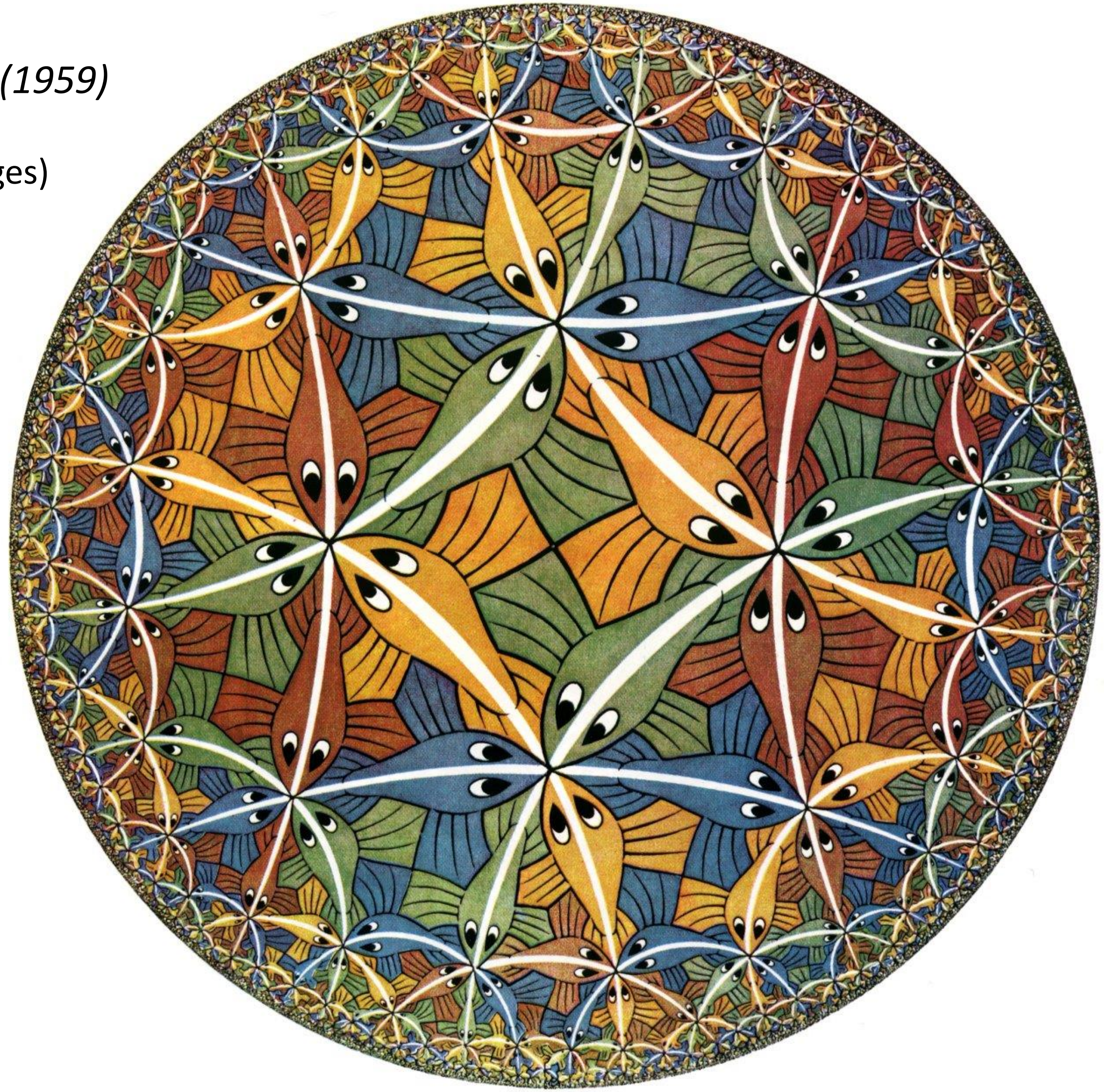
TITLE	*Snakes*
CREATION DATE	1969
MEDIUM	Woodcut printed in orange, green and black
SIZE	18 5/8 in 47.3 cm

AUCTION HOUSE	Sotheby's New York
AUCTION DATE	Apr 26 - Apr 27, 2012
ESTIMATE	\$30,000 - 40,000
SALE	\$80,500

Visit the lot on sothebys.com
(NOTE: Auction houses occasionally remove older lots from their websites)



M.C. Escher
Cirkellimiet III (1959)
Woodcut
(20 printing stages)

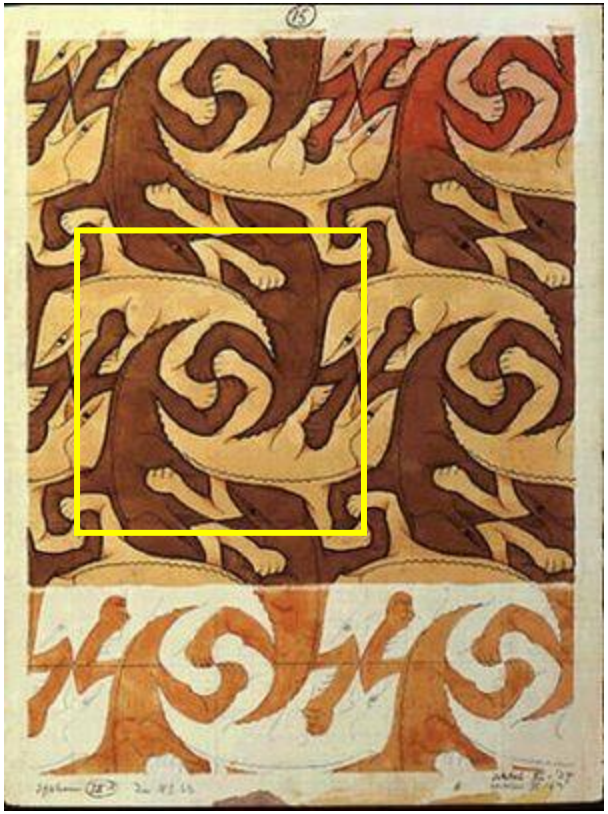


Computers 1960 's

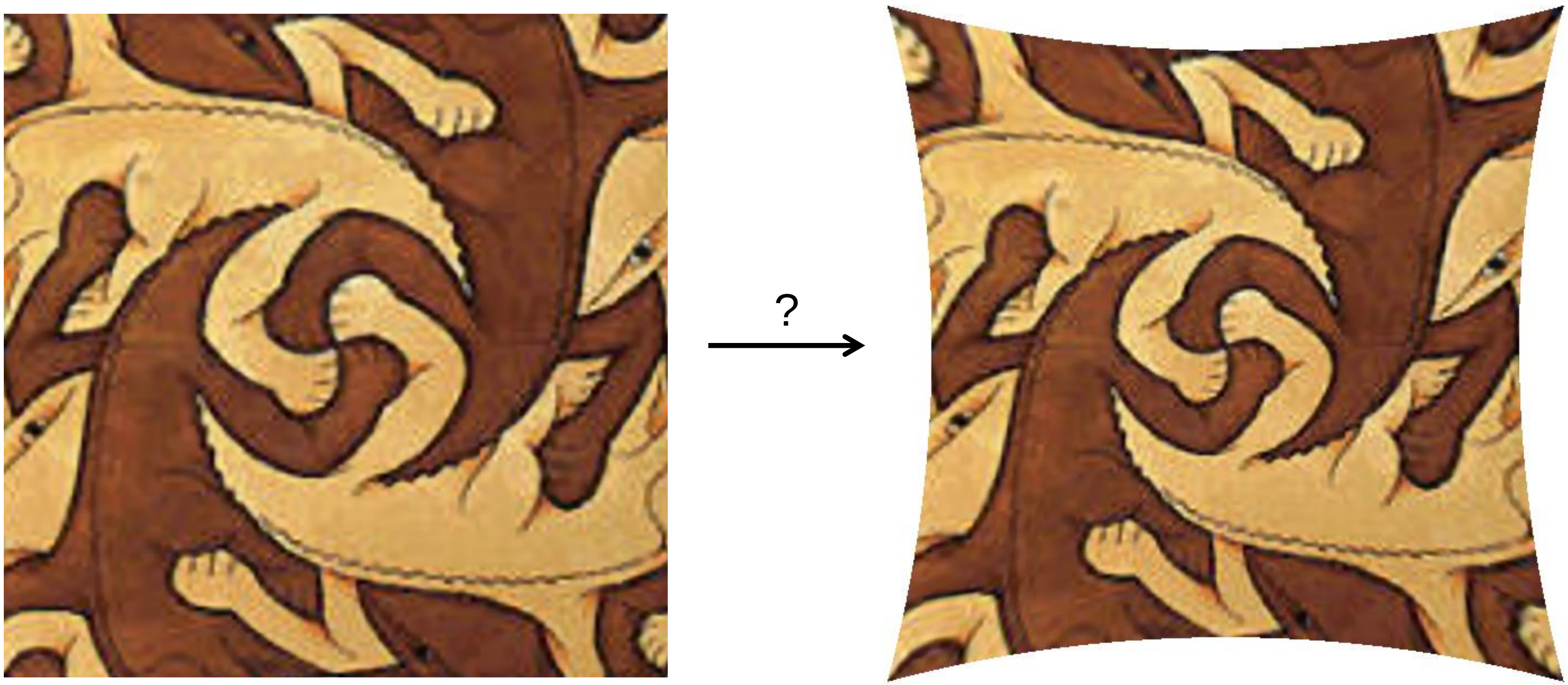
Computers 1960 ‘s

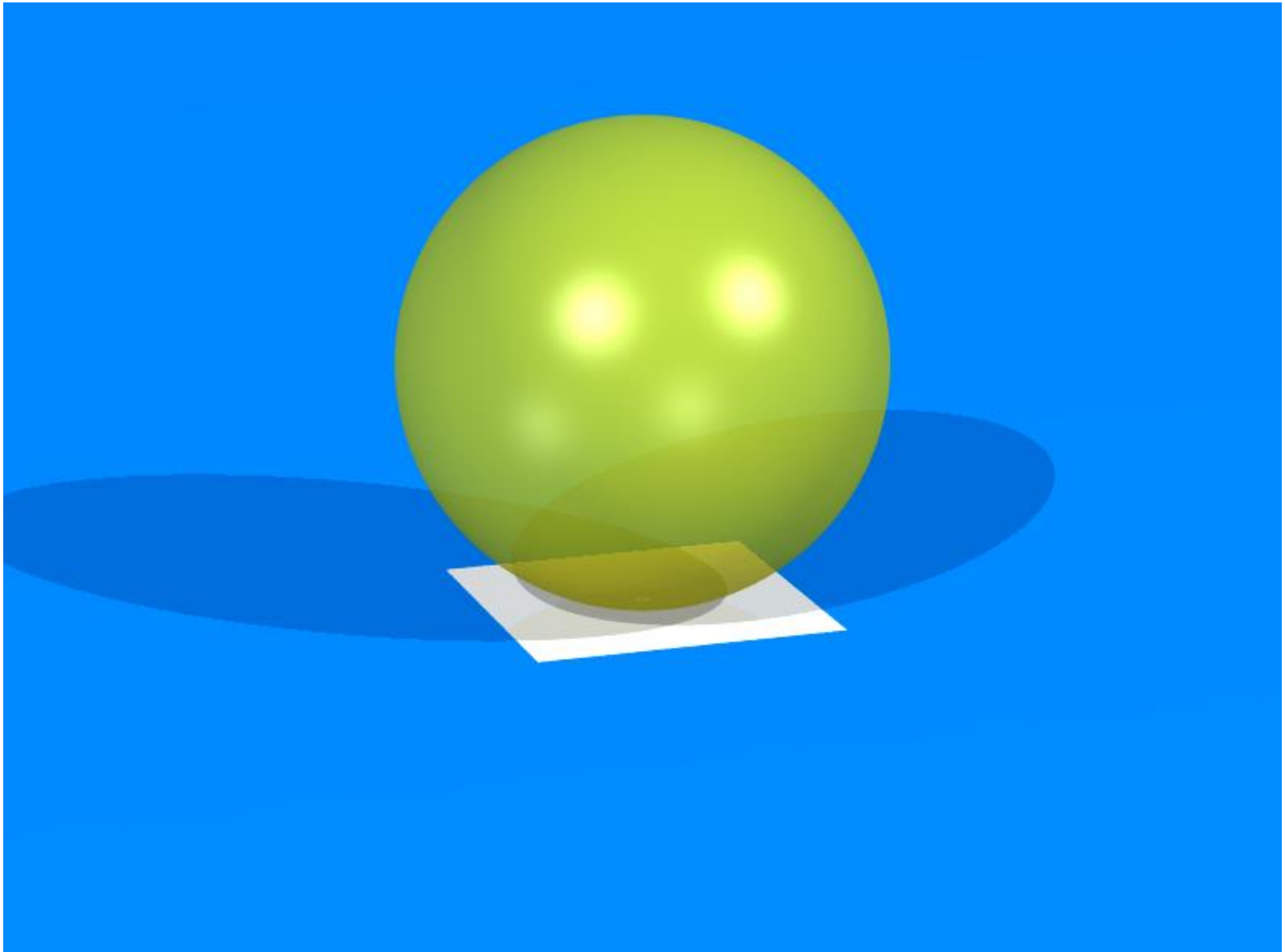
Hard disc 5 Mb

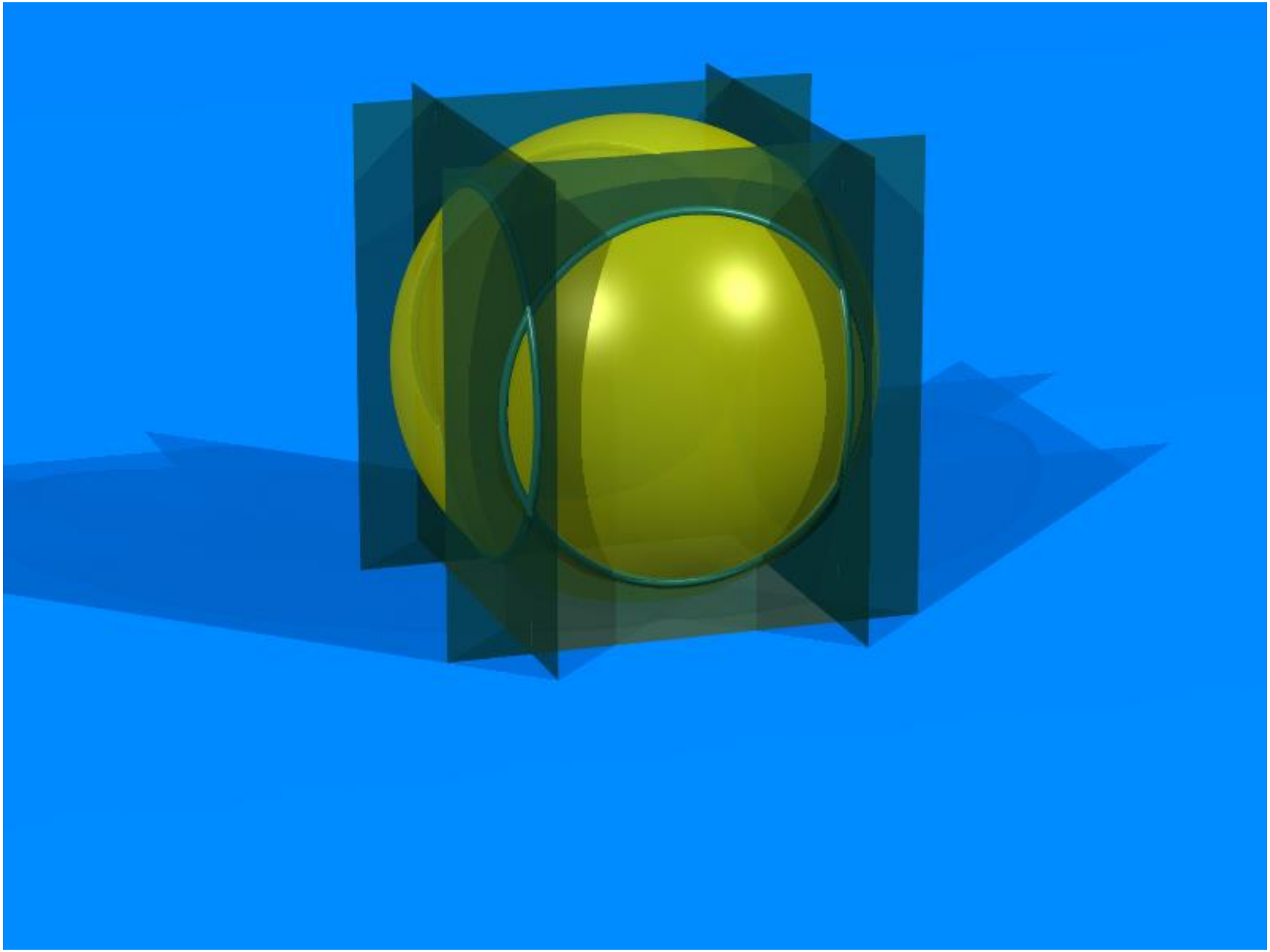
‘Hyperbolisation’

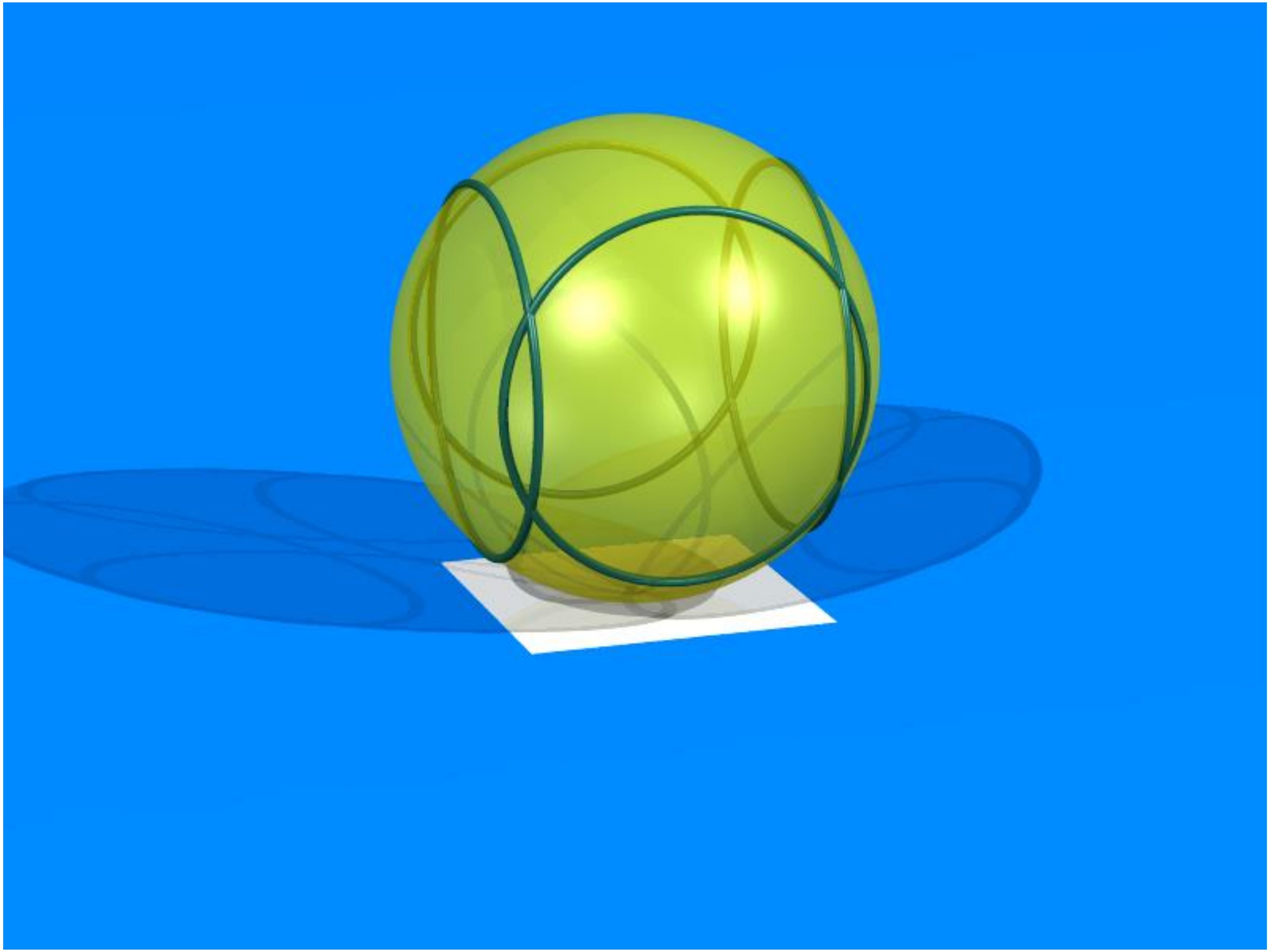


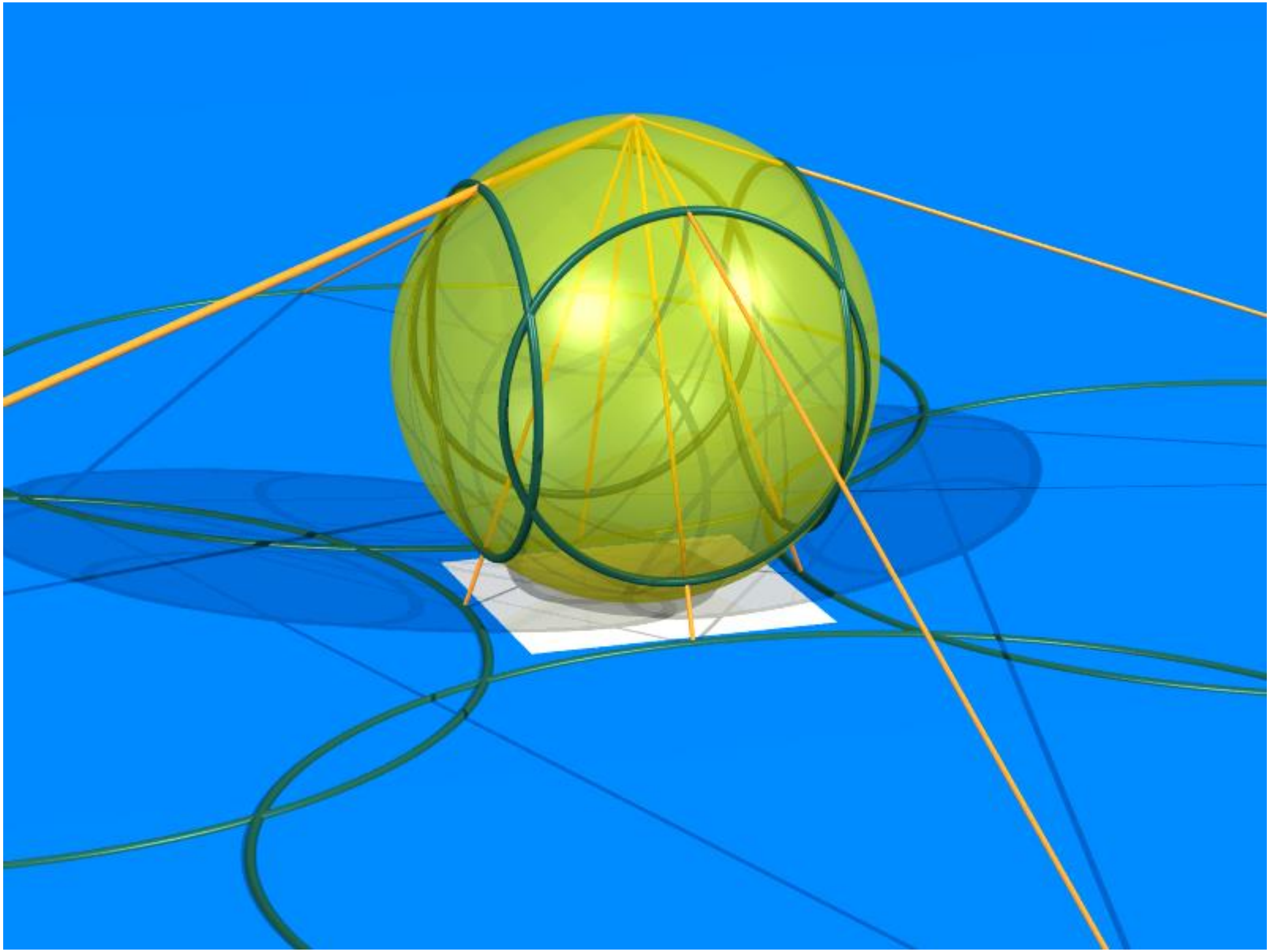
- Creating a hyperbolic version using one fundamental tile

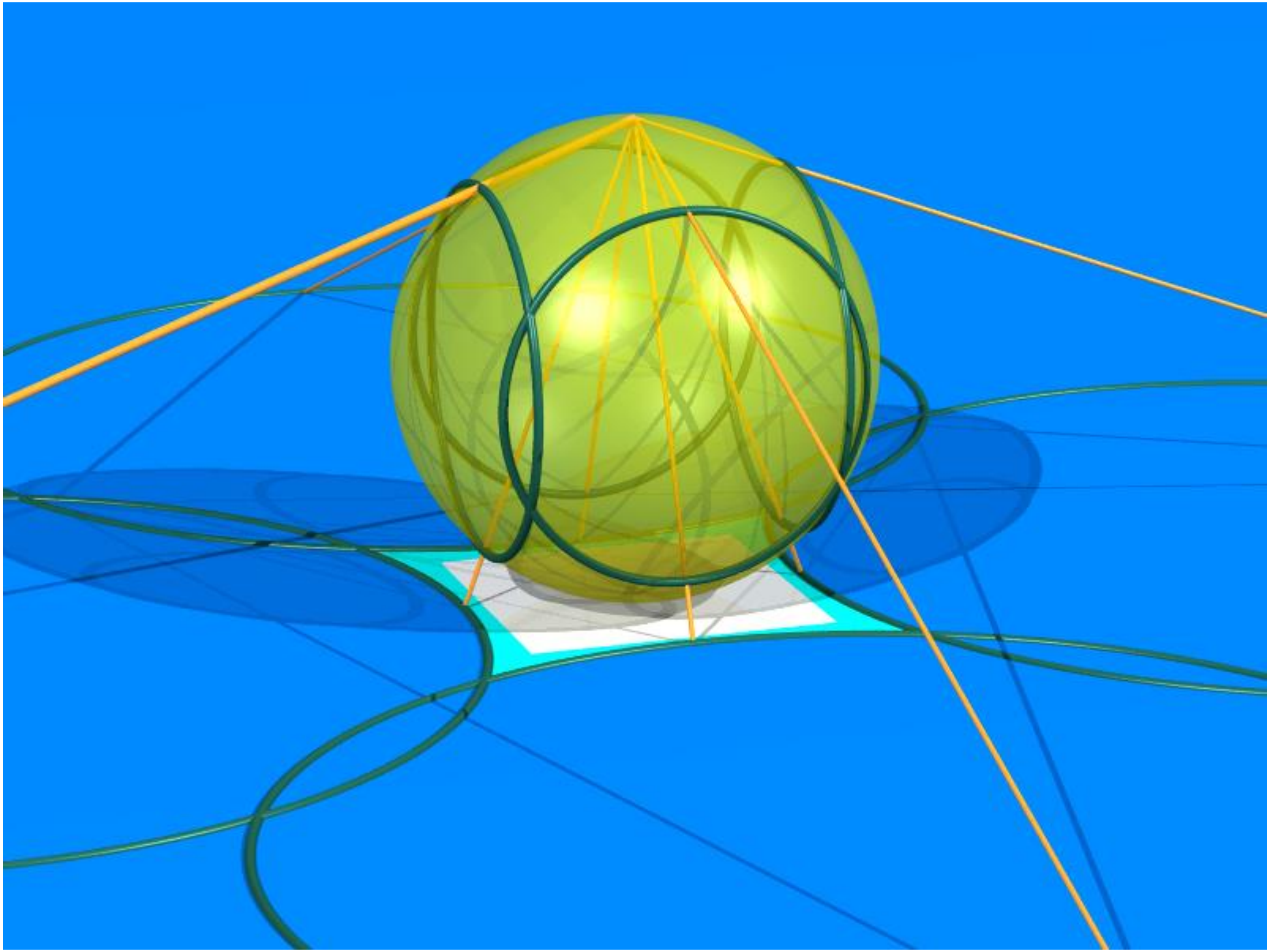


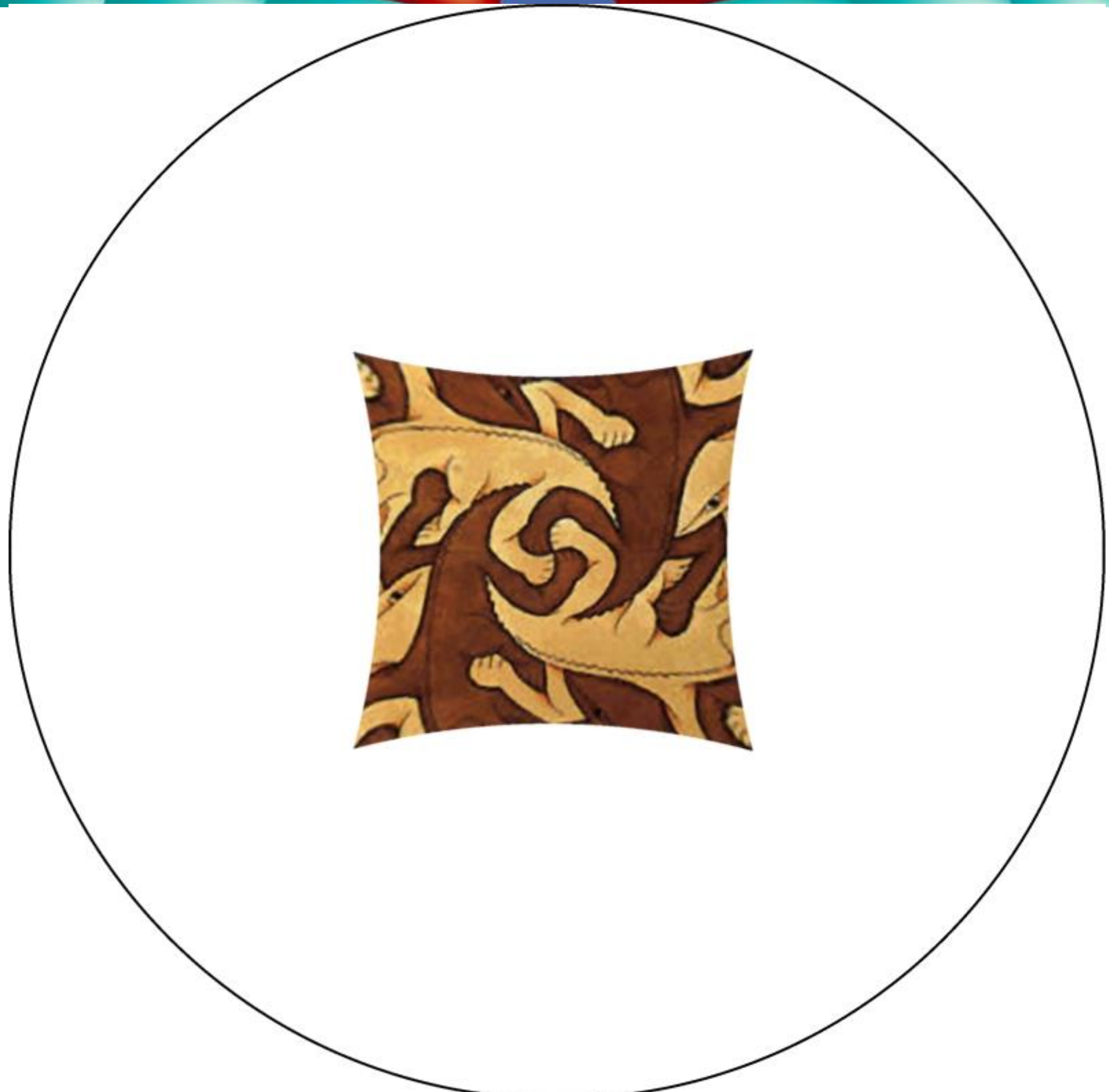


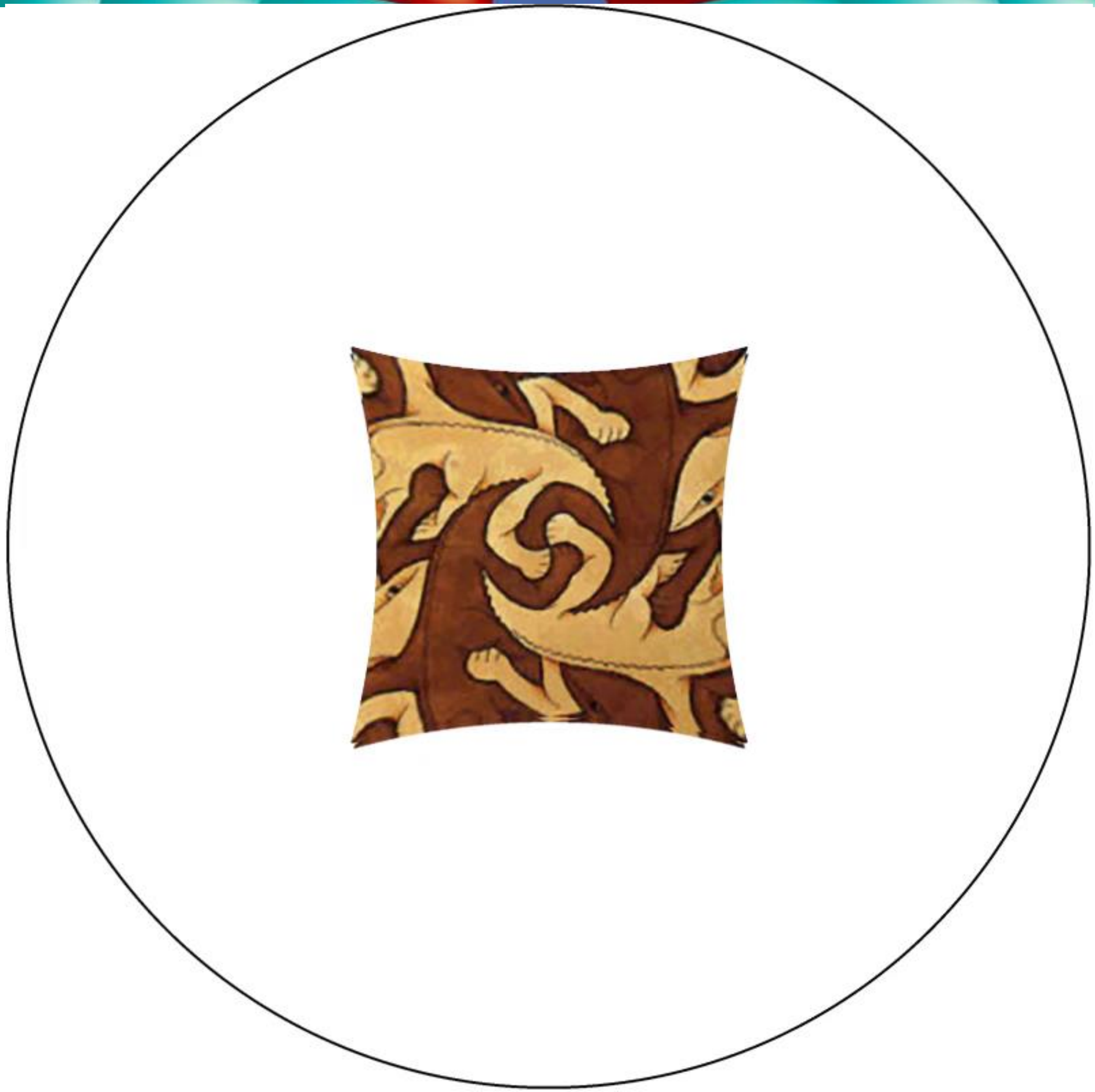


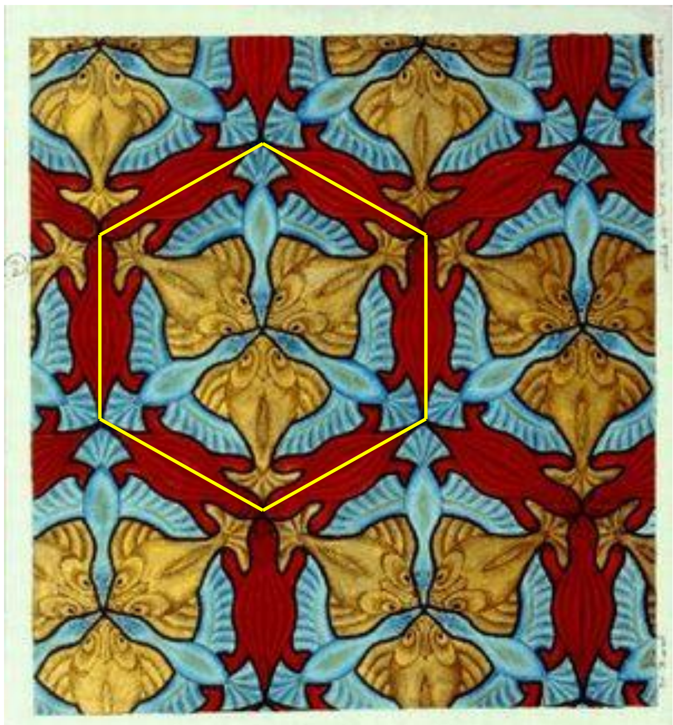


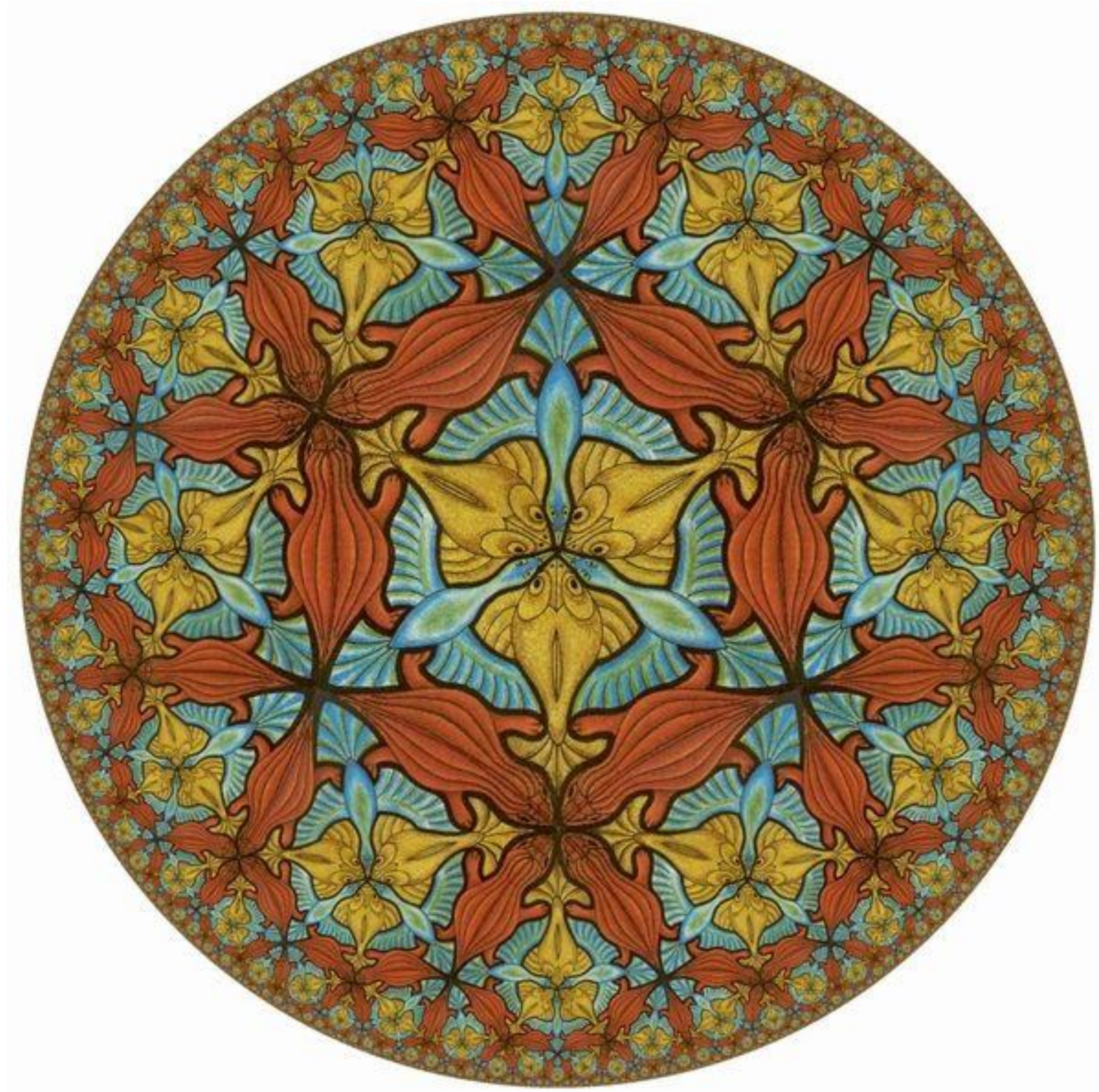


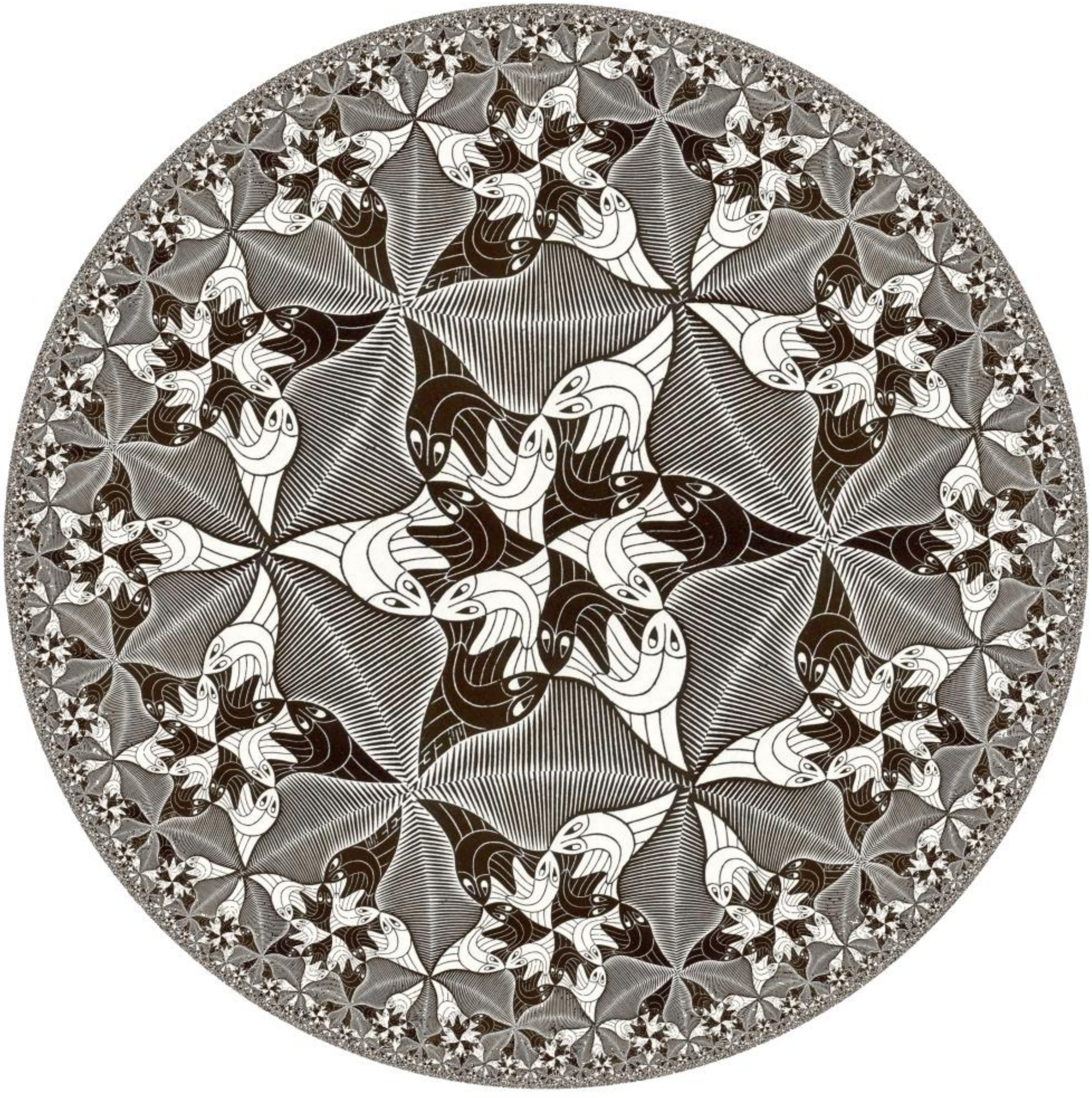


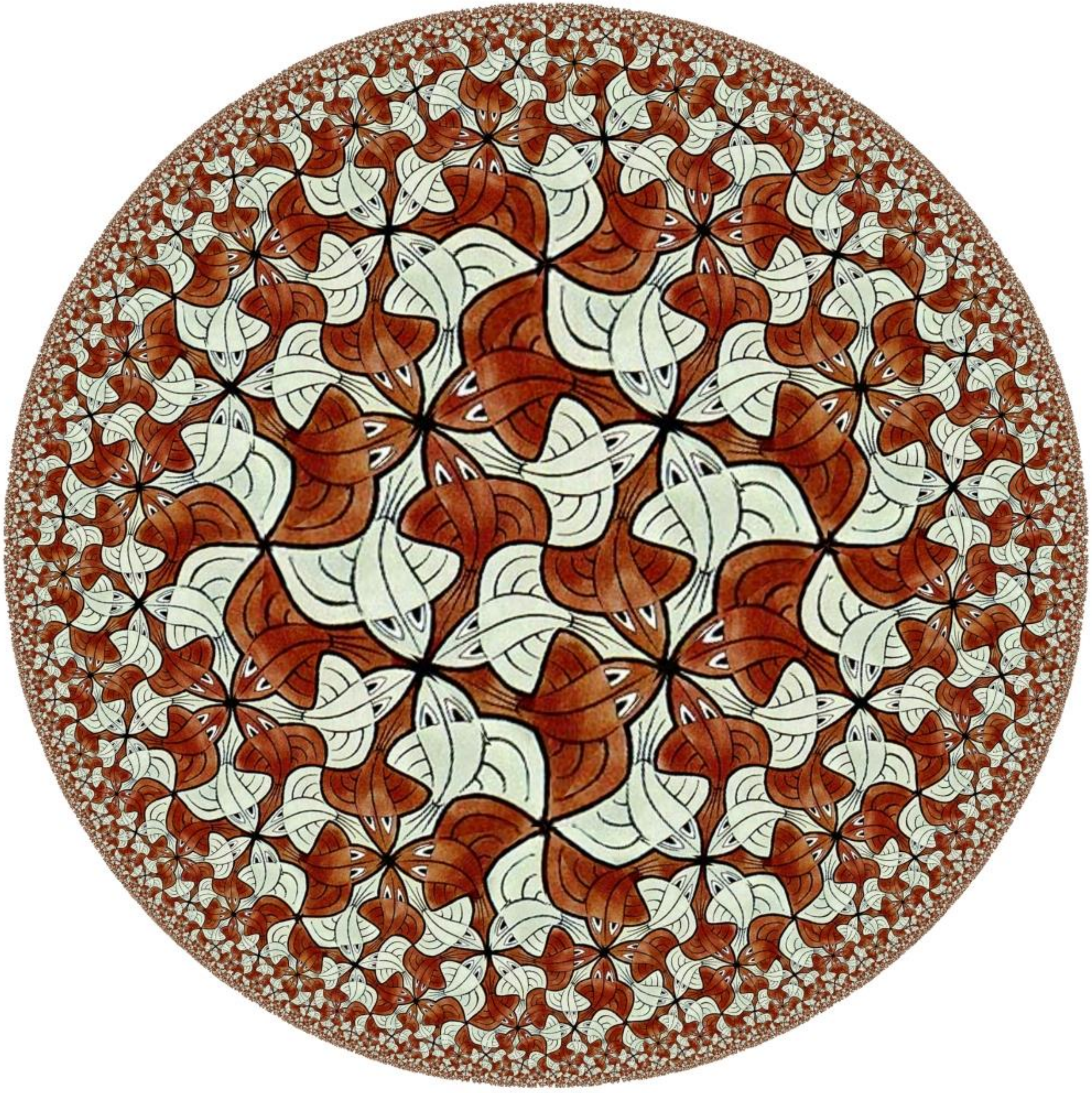


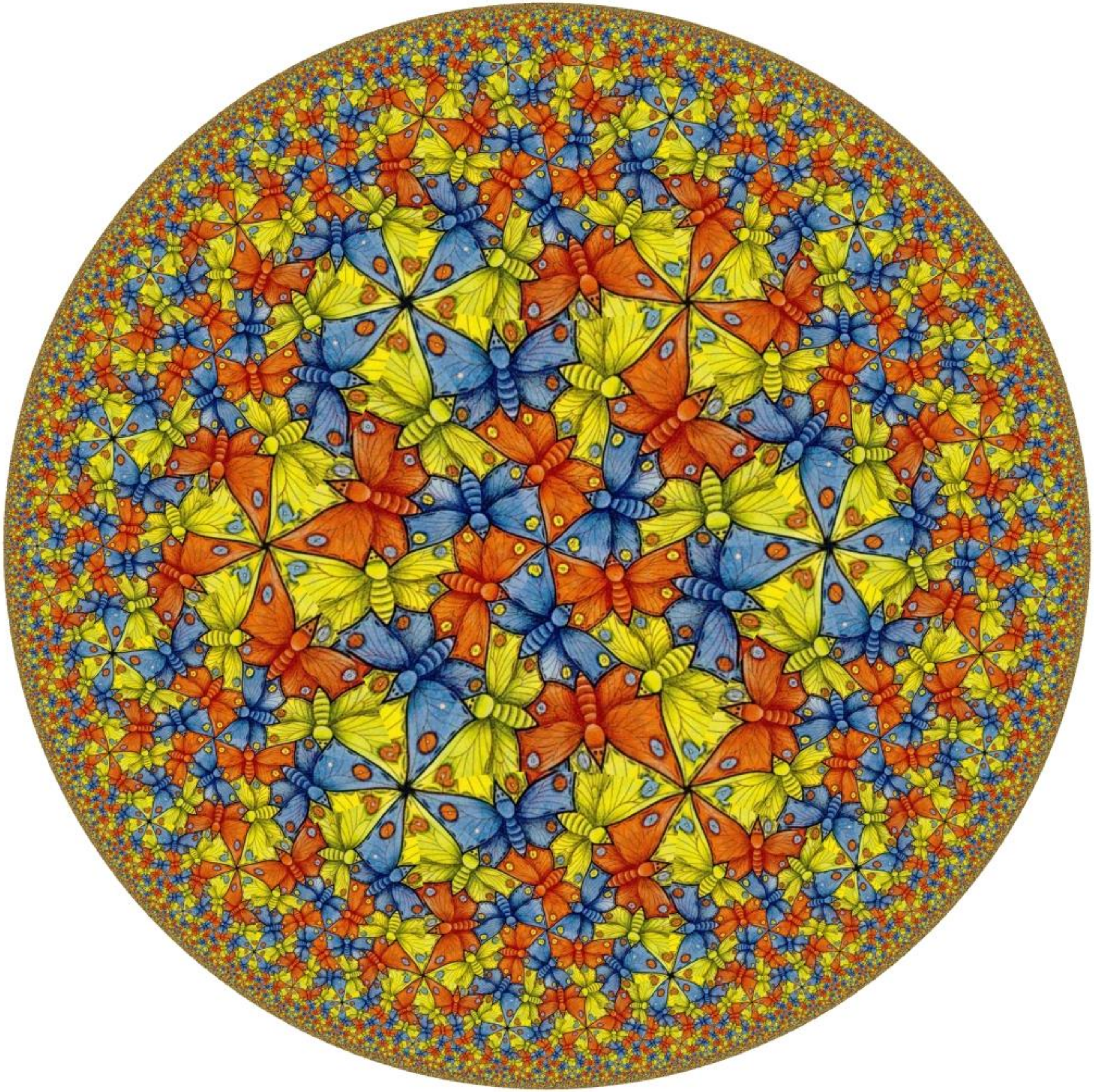


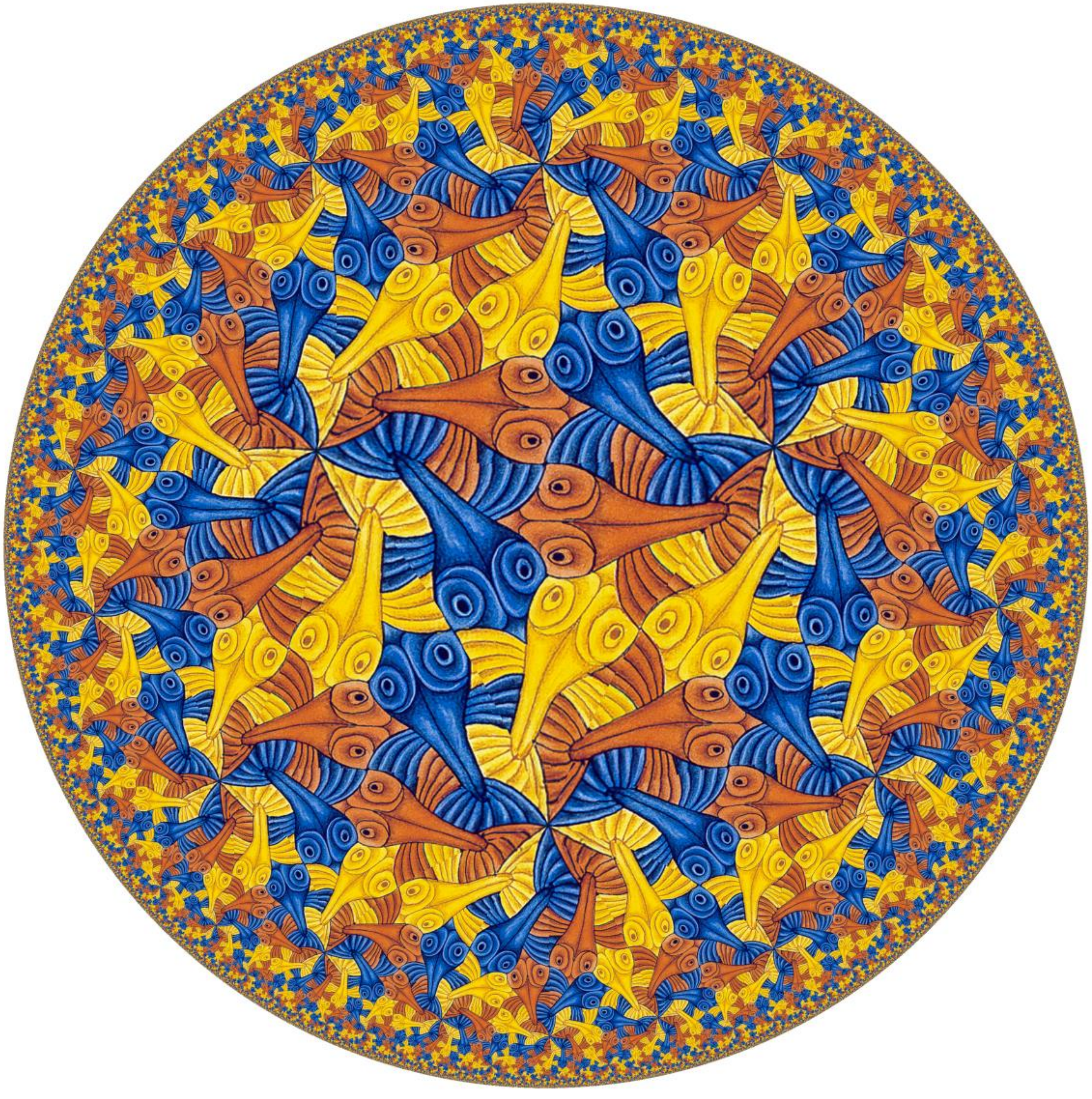


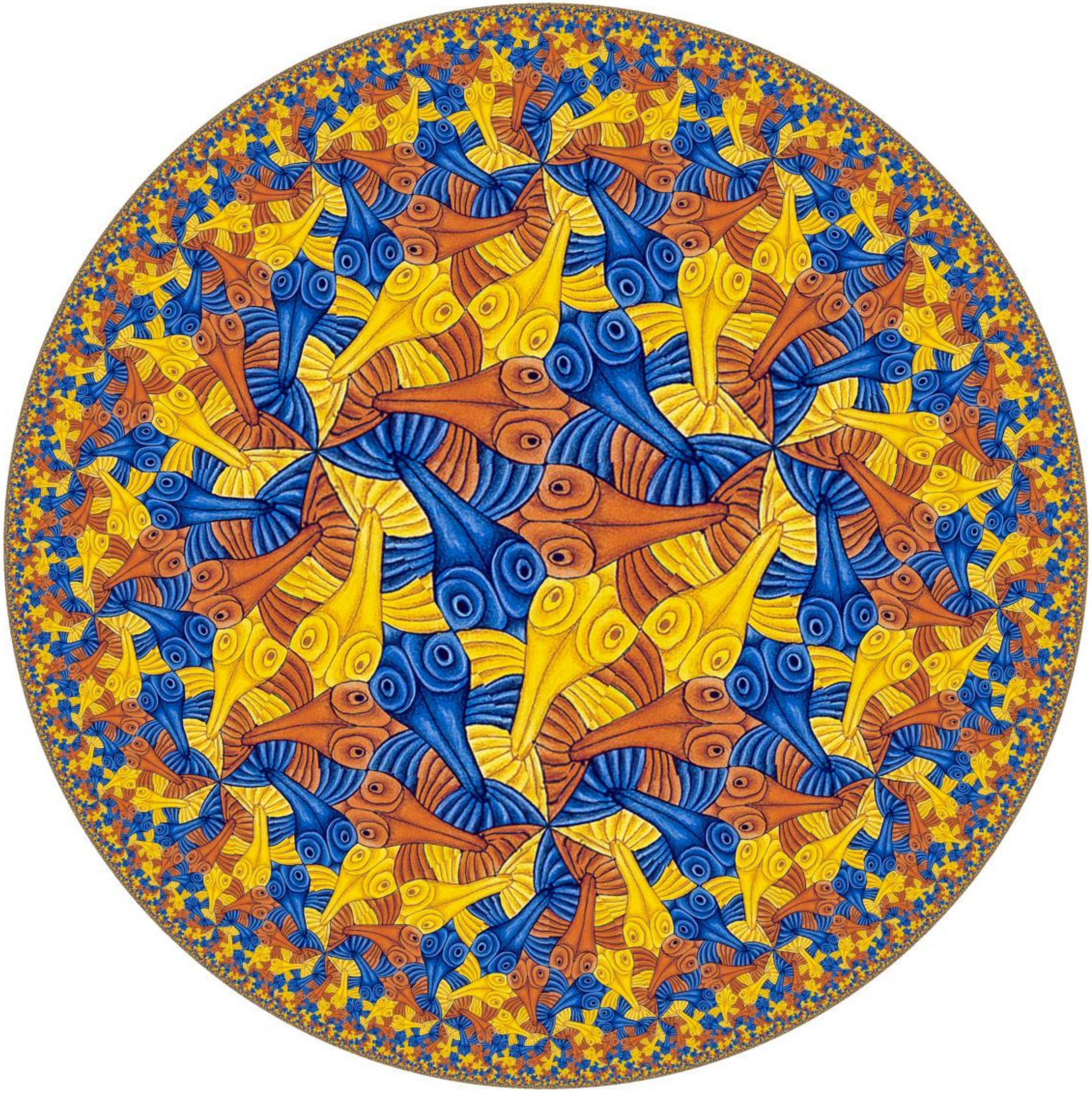




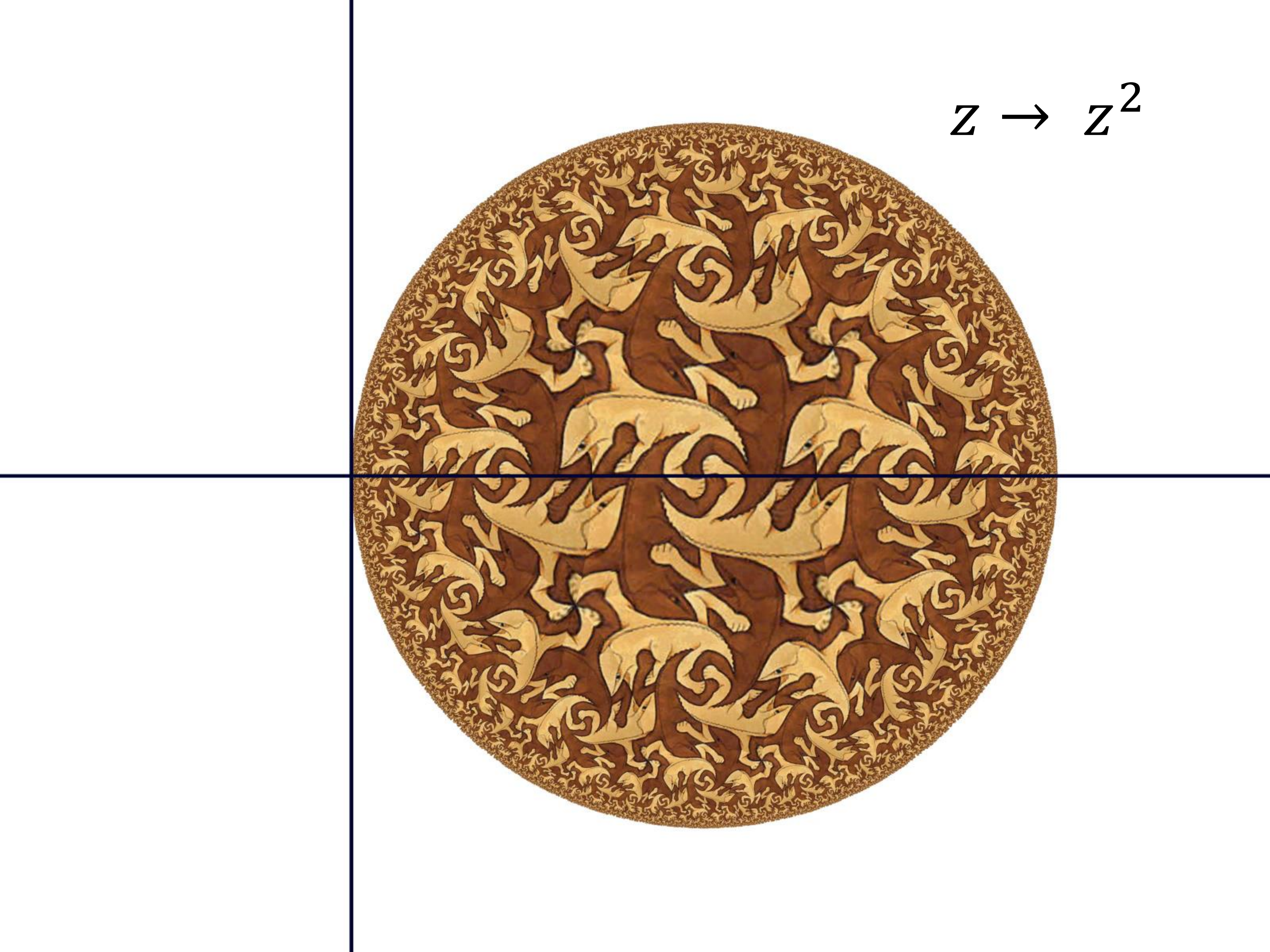




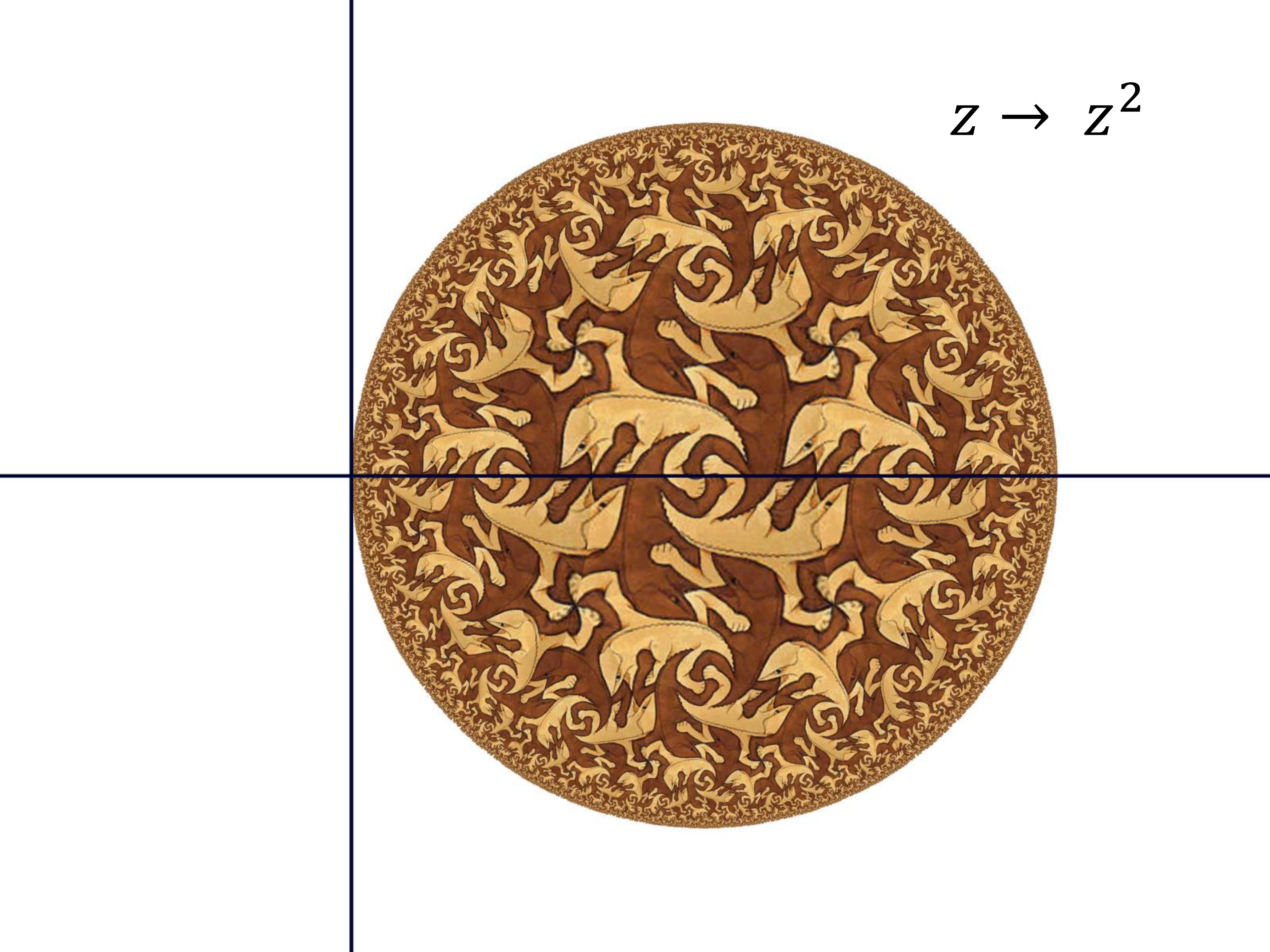




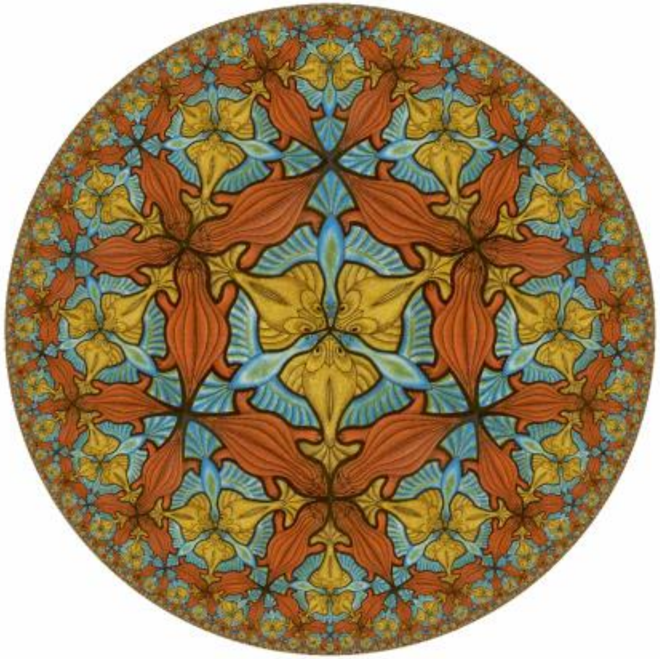
$z \to z^2$



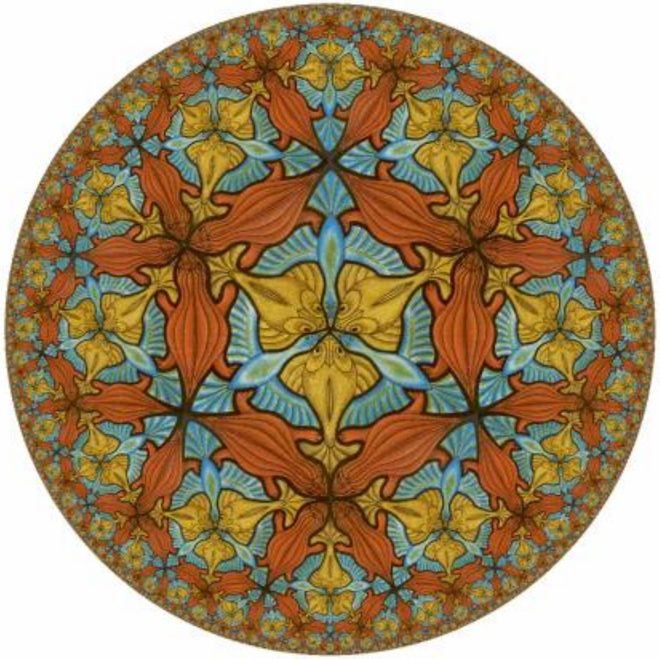
$$z \to z^2$$



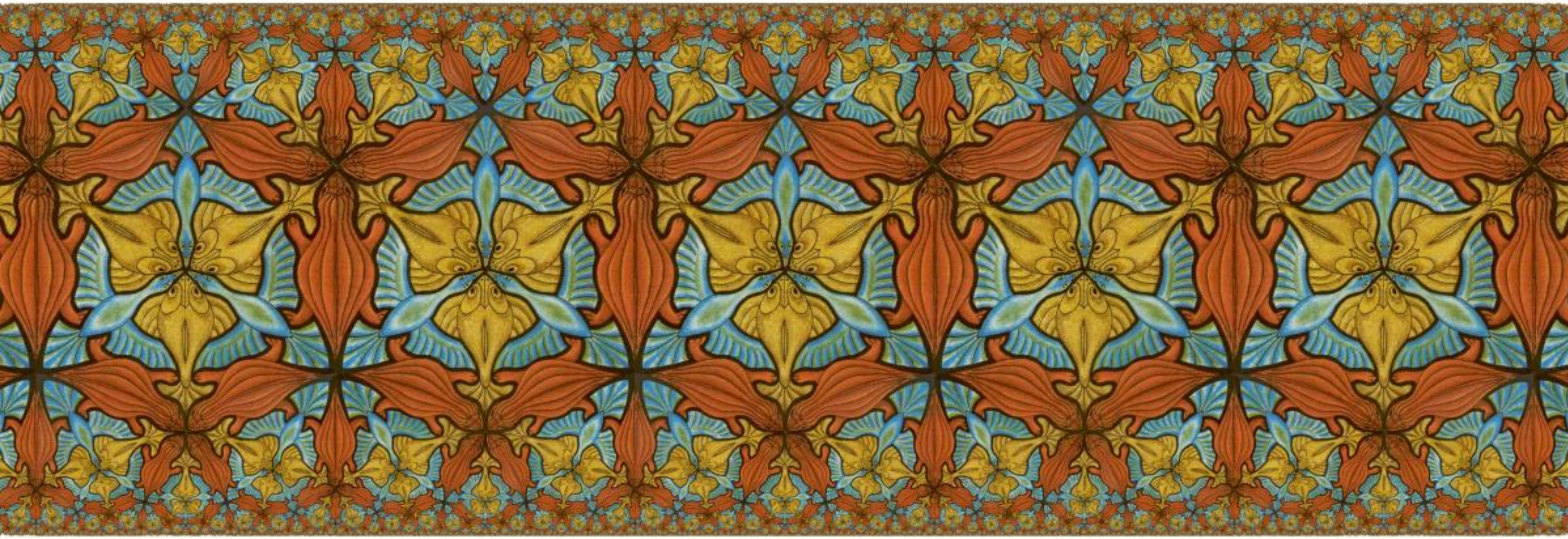
$$z \rightarrow \frac{2}{\pi} \ln\left(\frac{1+z}{1-z}\right)$$

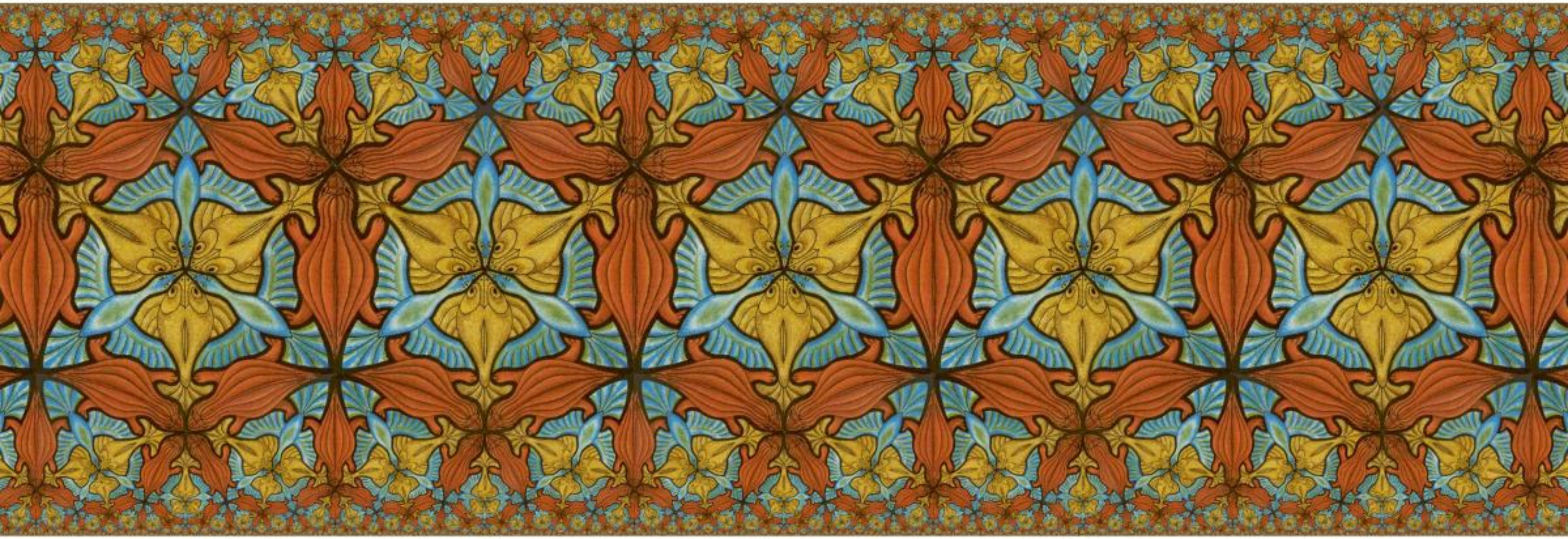


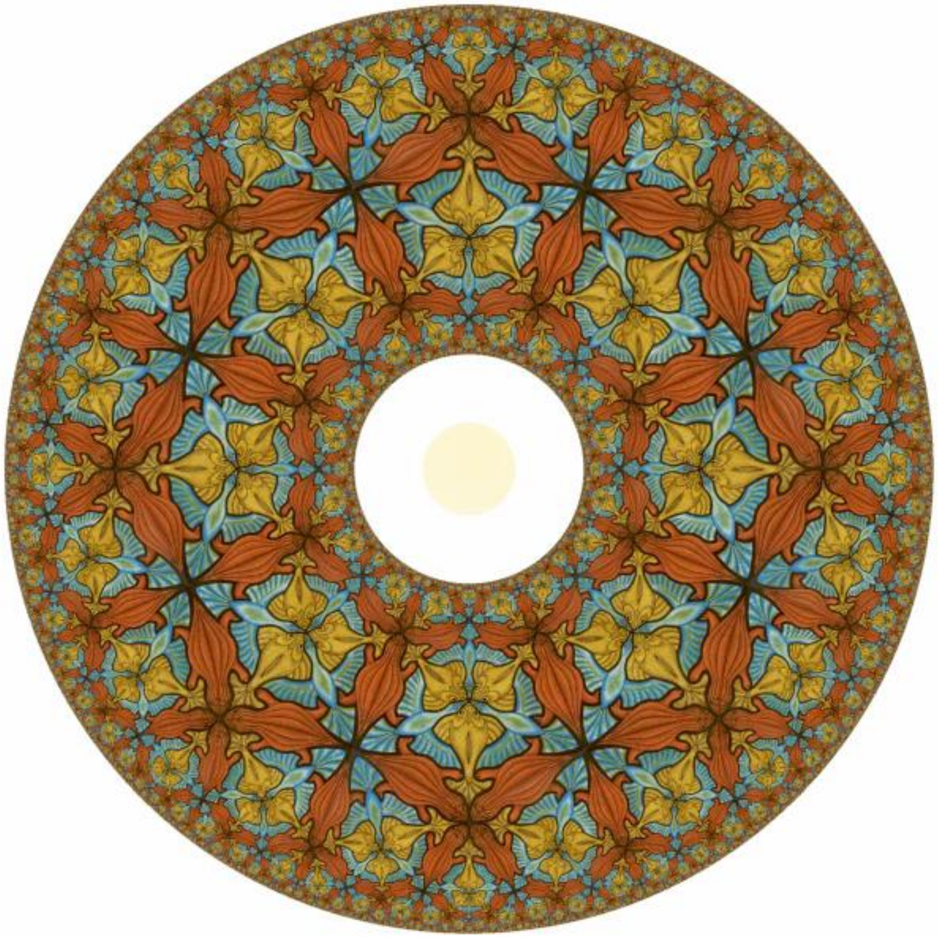
$$z \to \frac{2}{\pi} \ln\left(\frac{1+z}{1-z}\right)$$

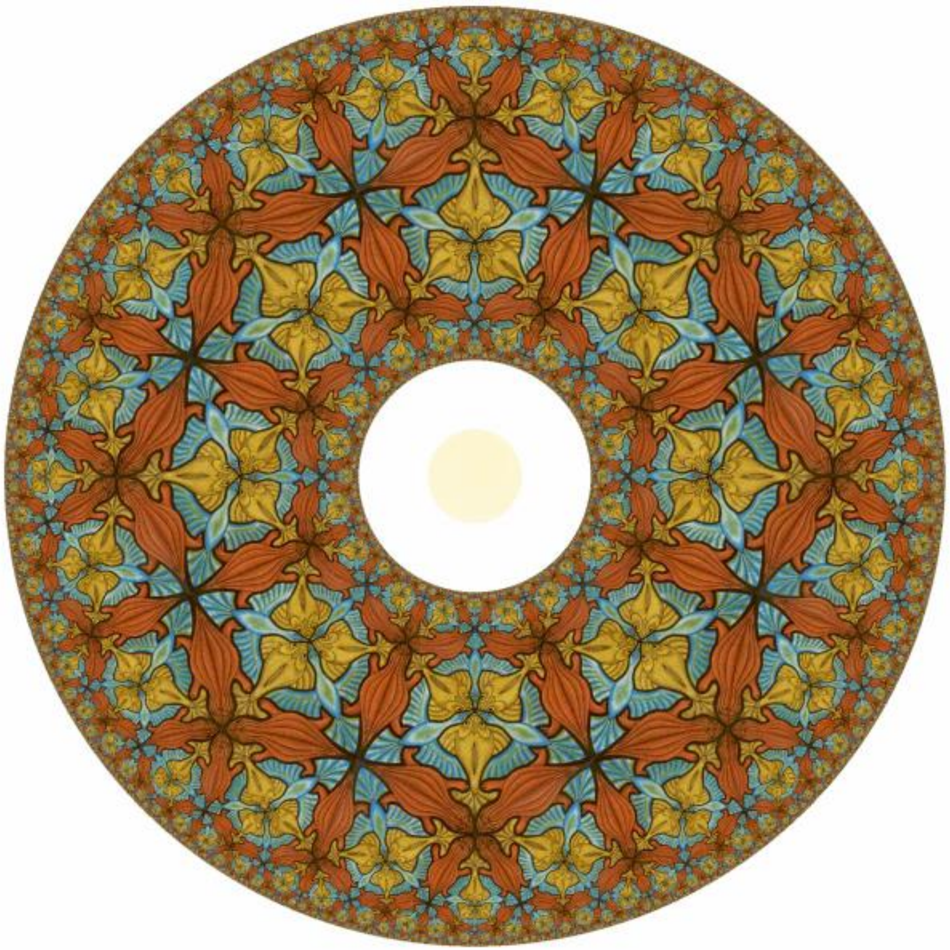


$$z \to e^{\frac{i2\pi(z+i)}{kP}}$$

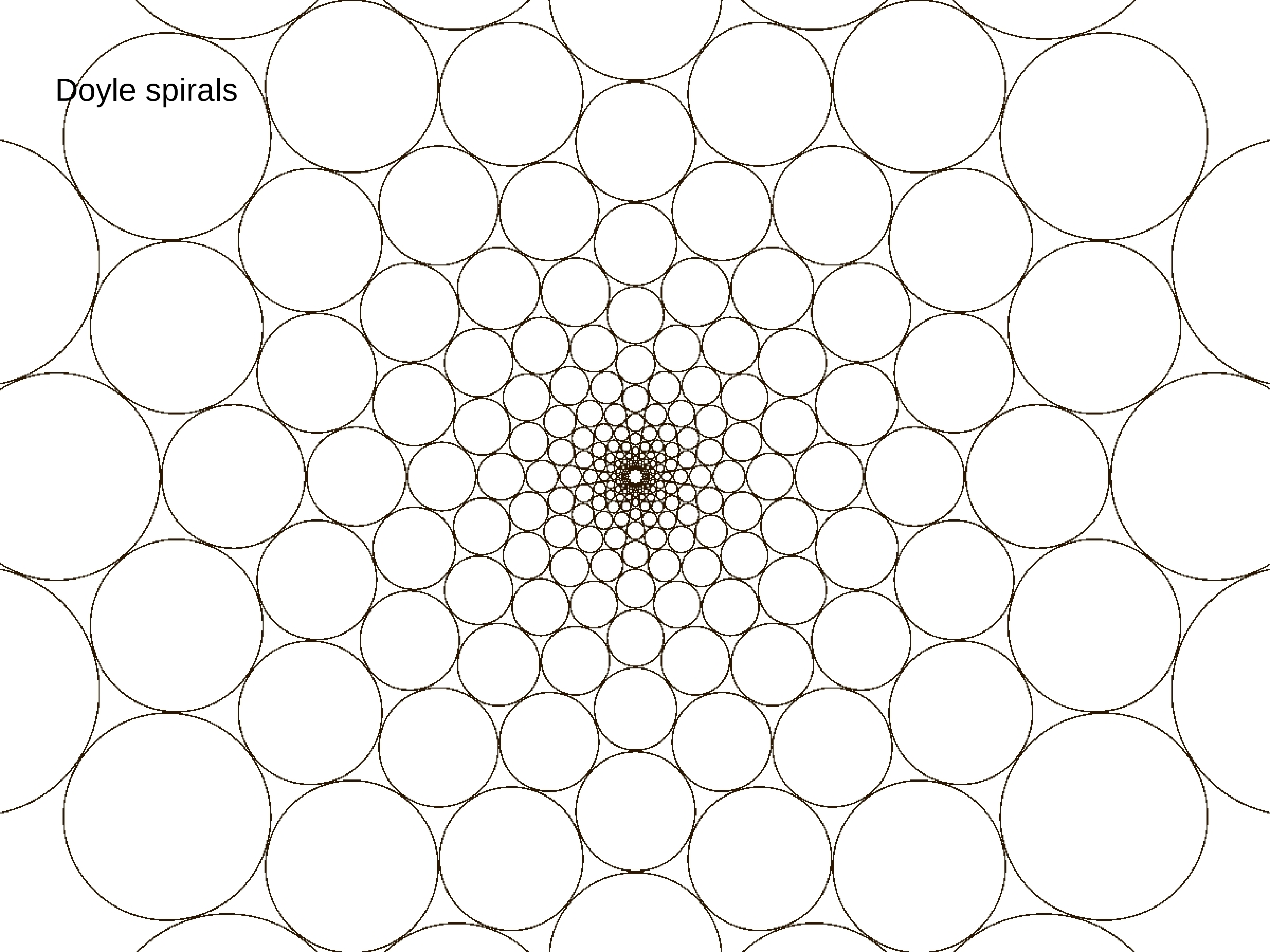


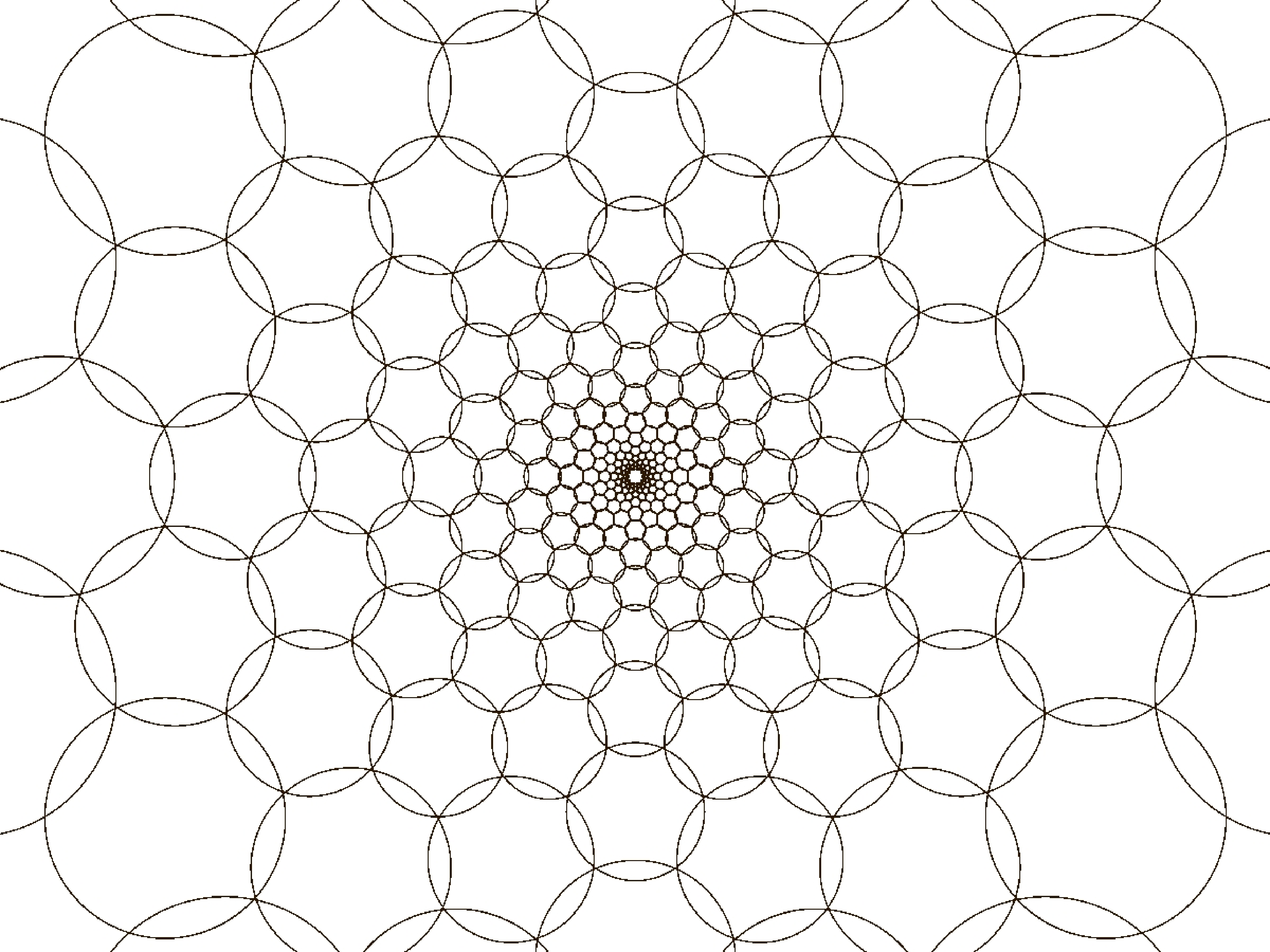


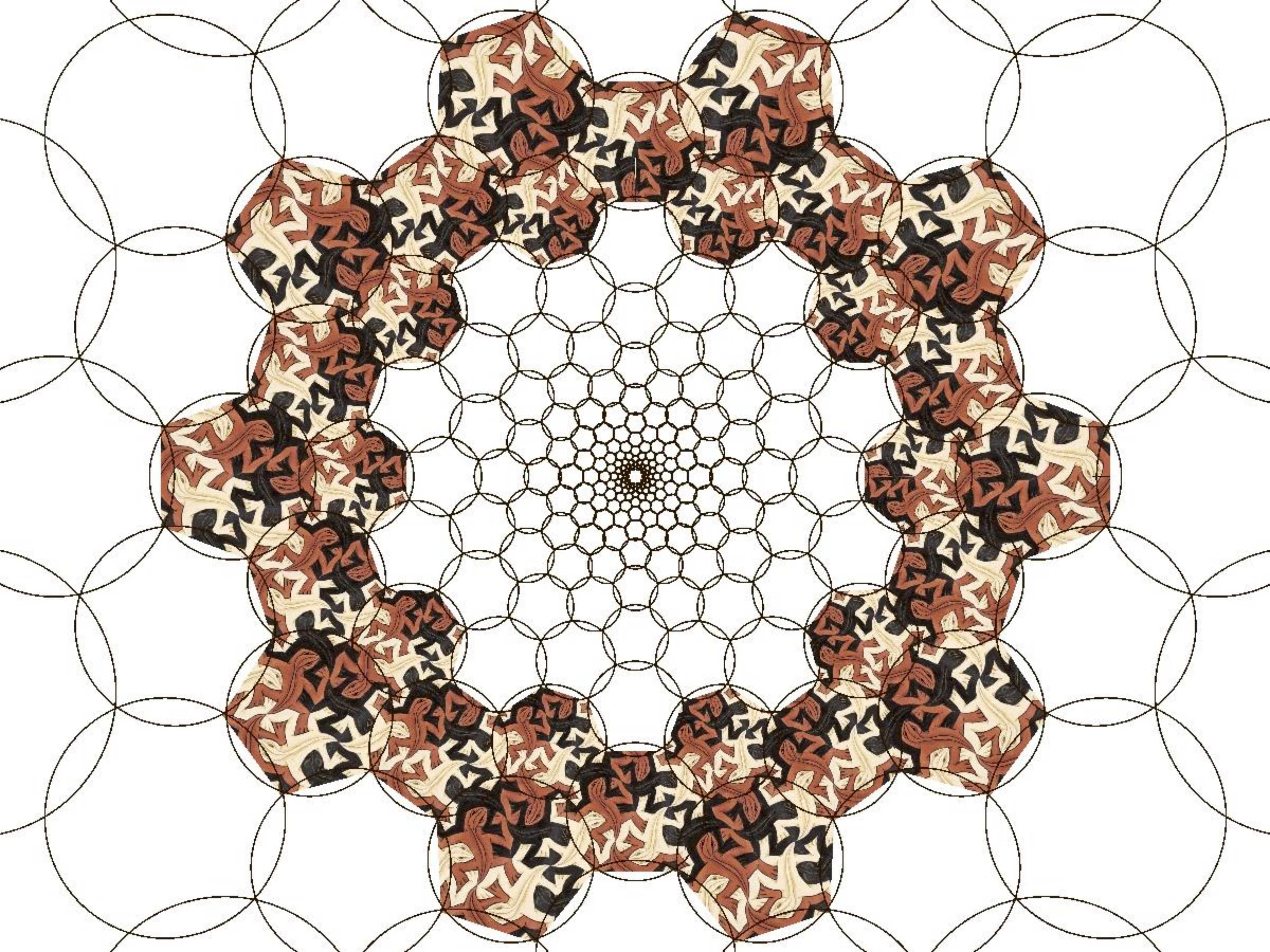


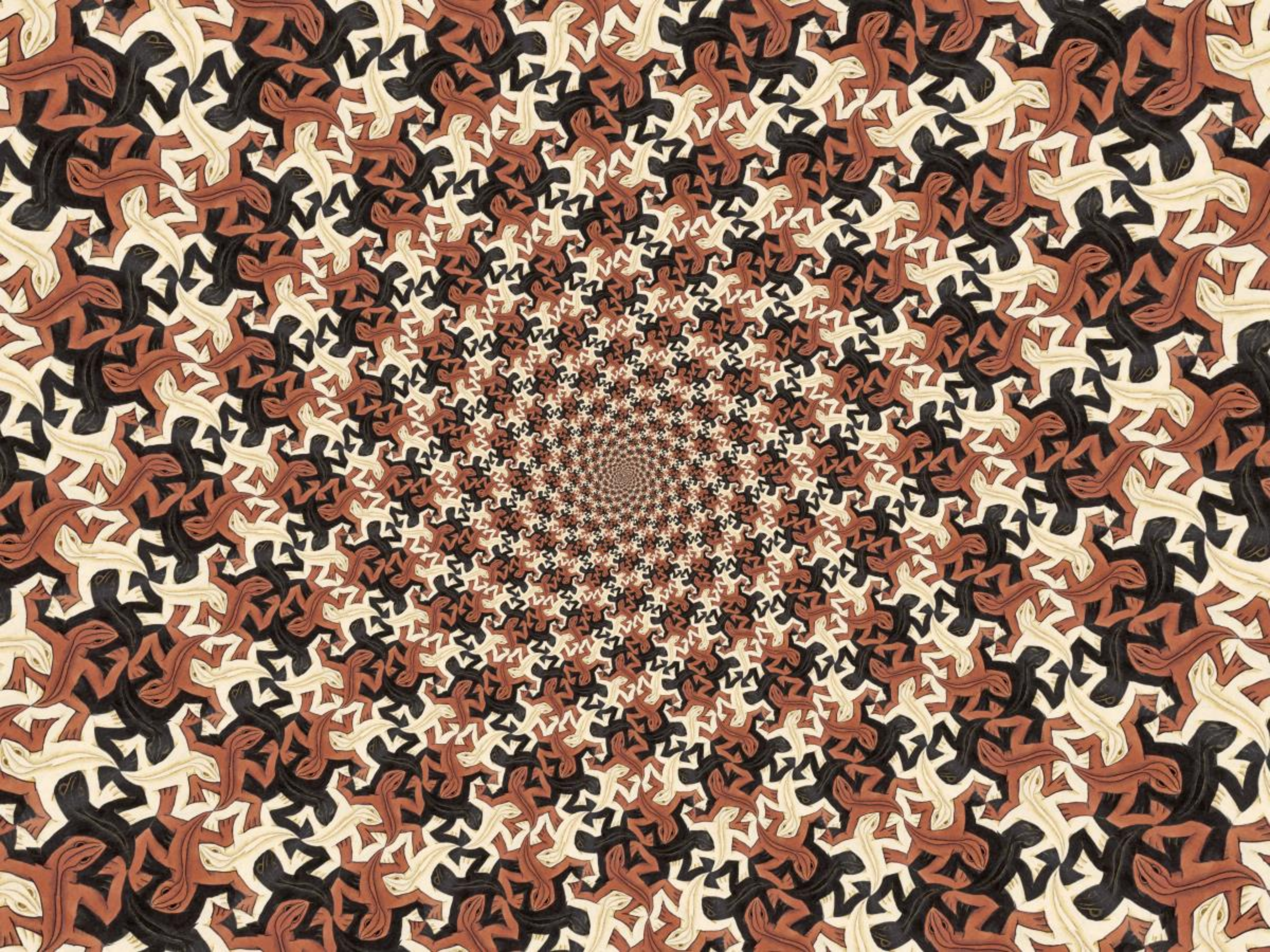


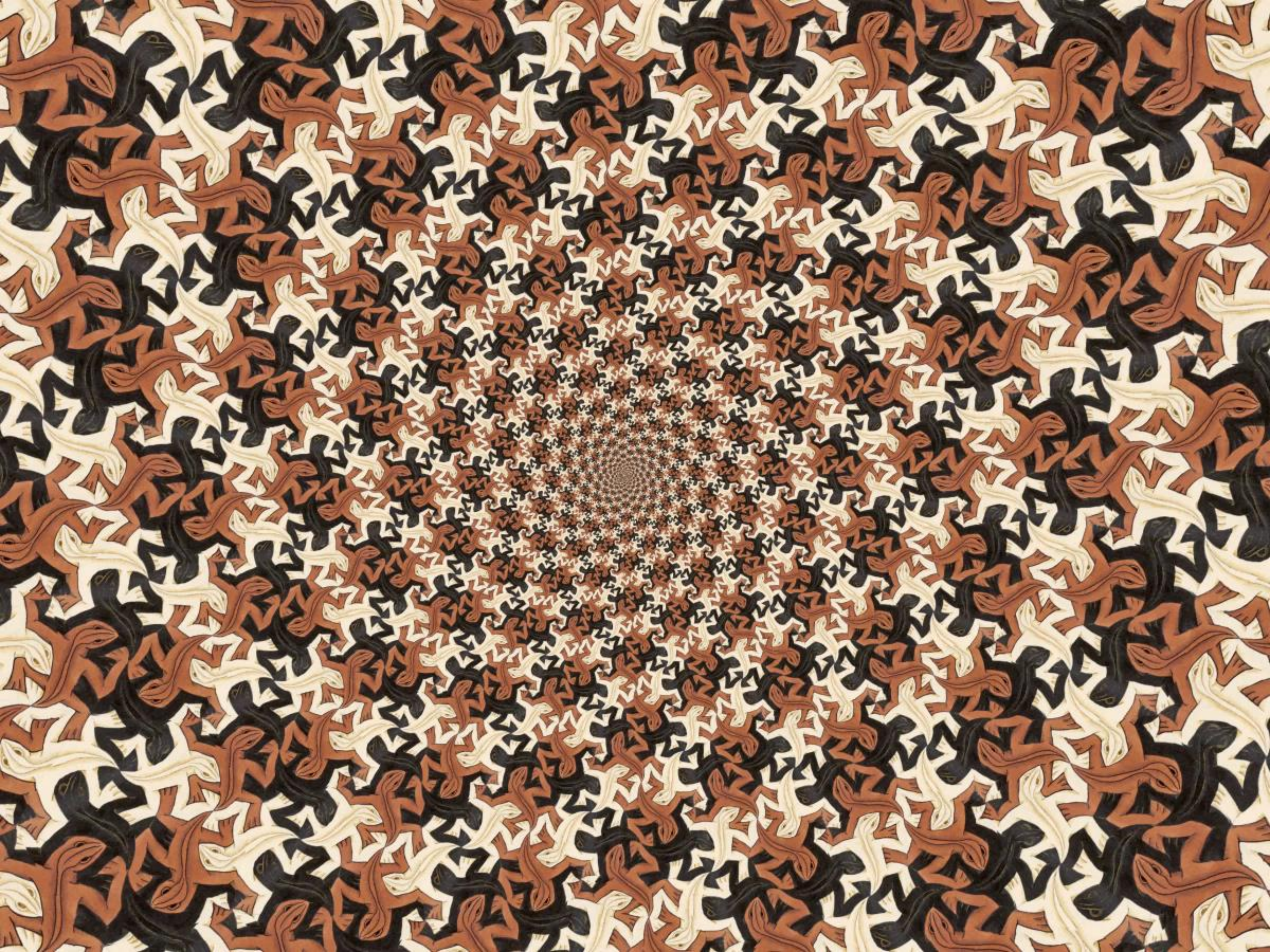
Doyle spirals

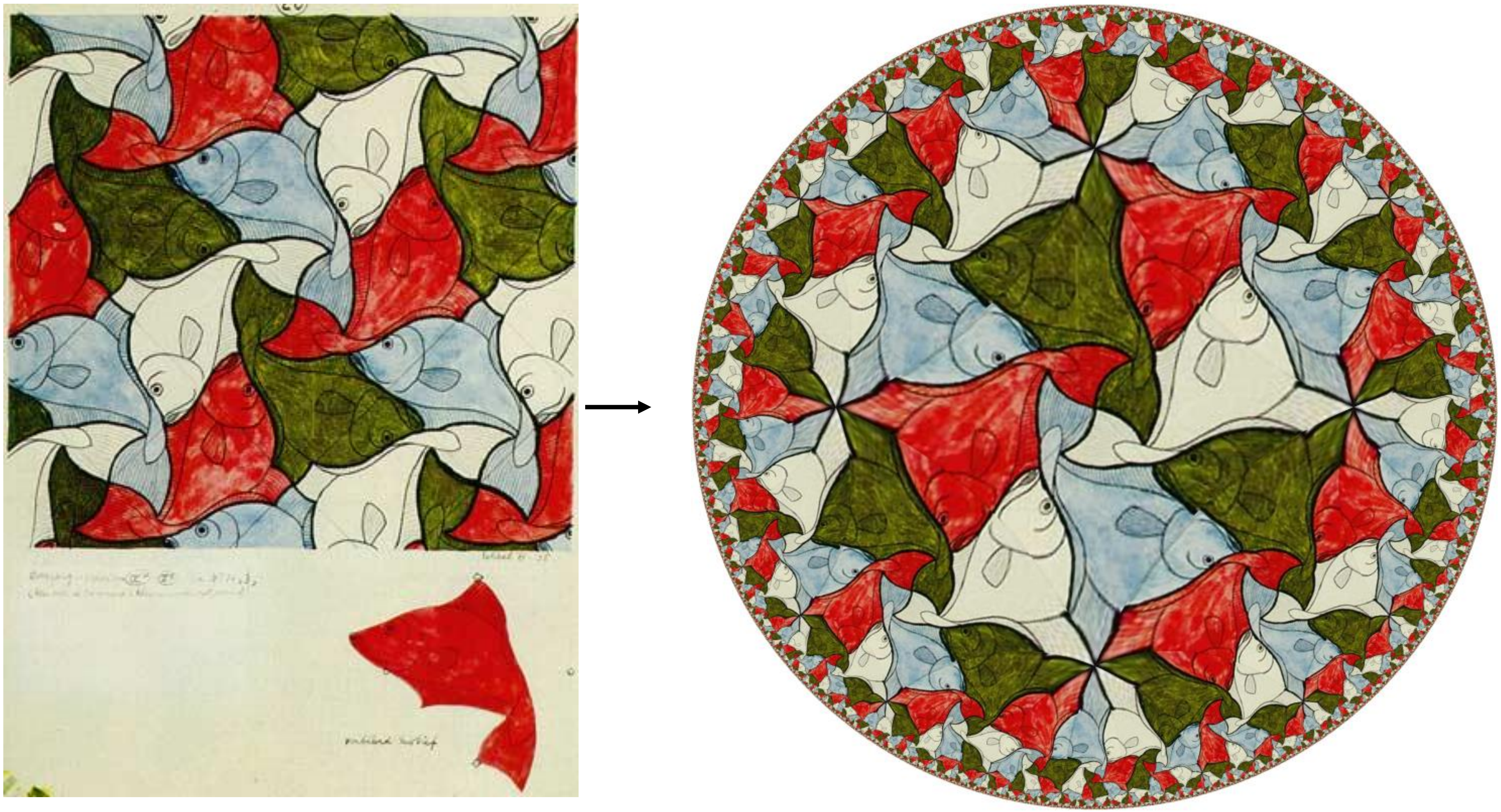


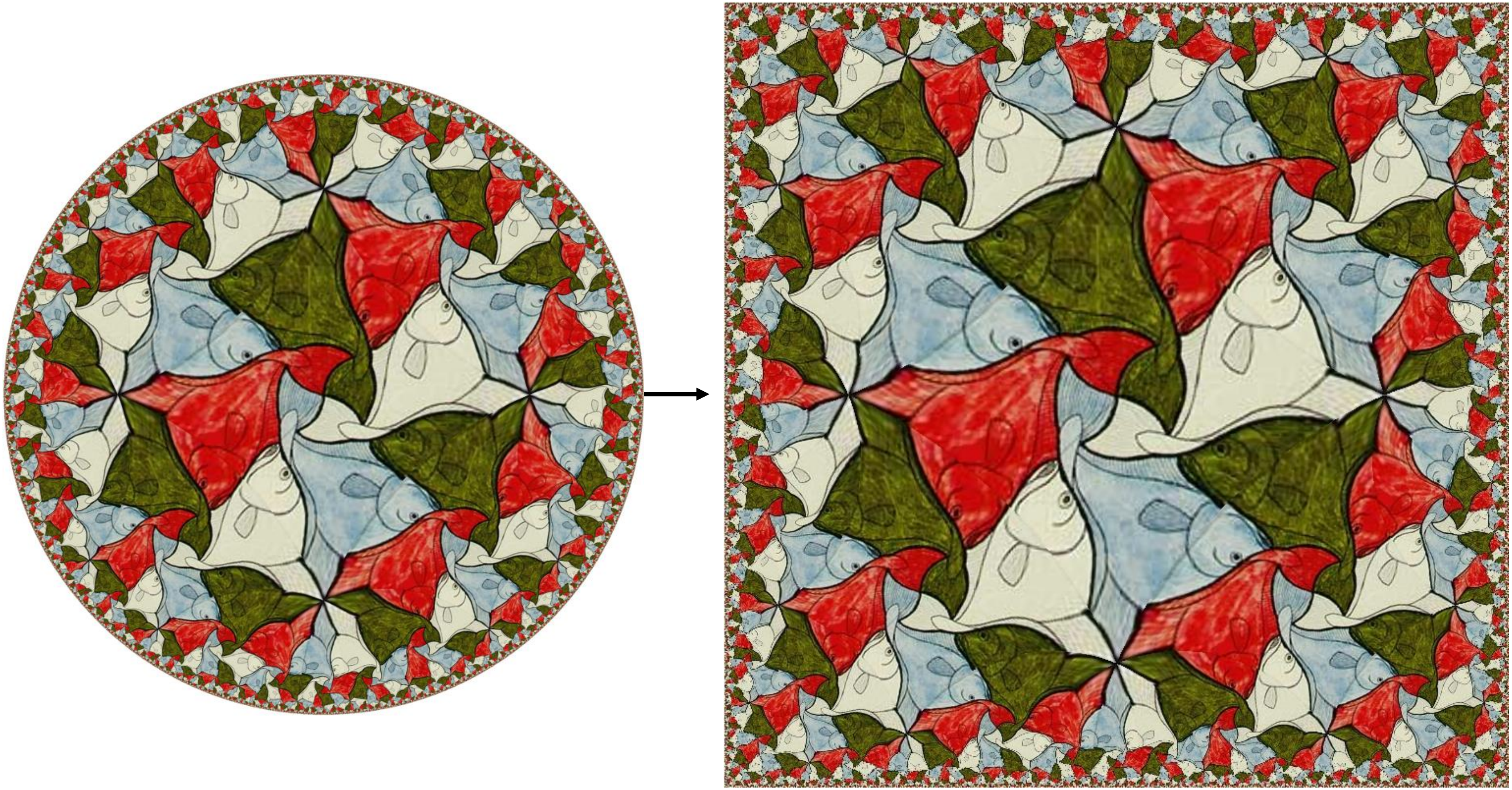




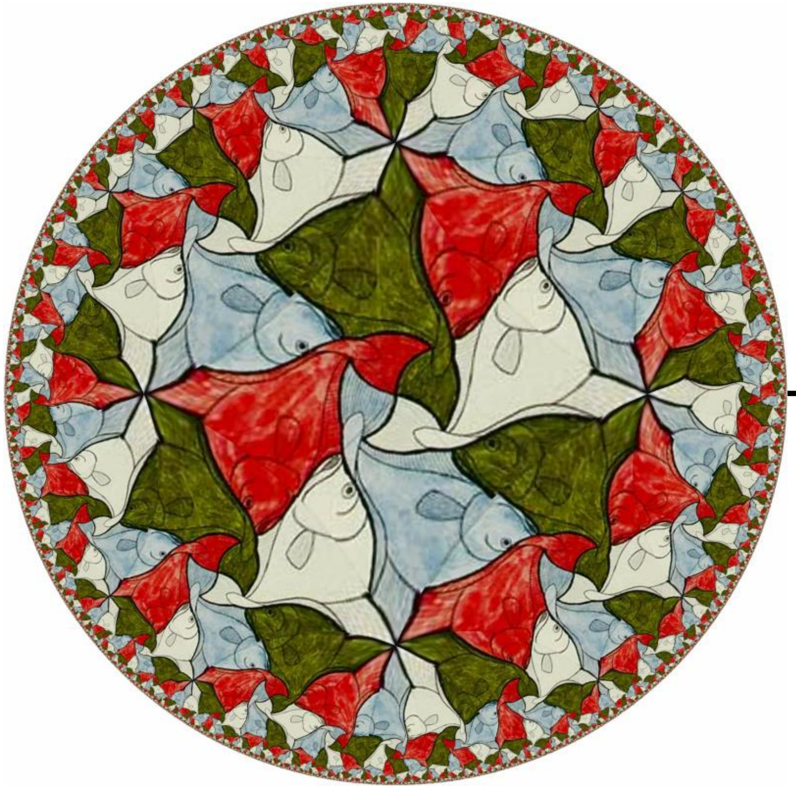


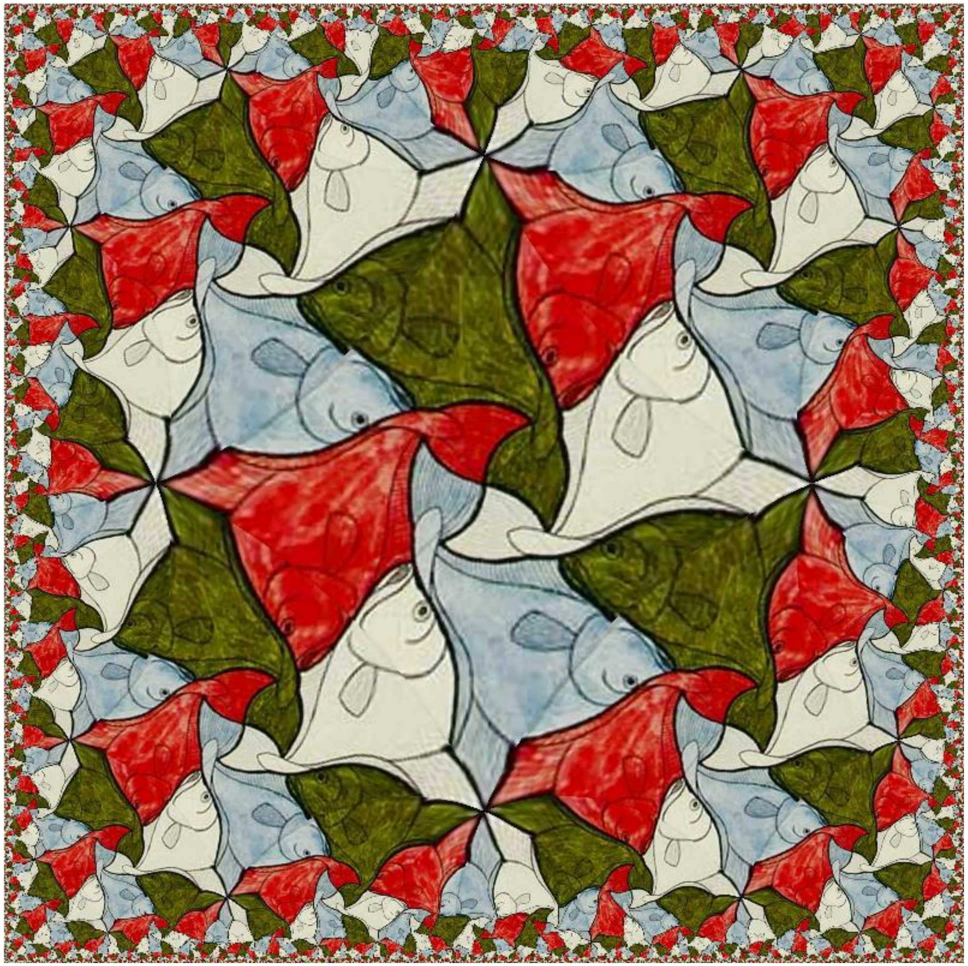






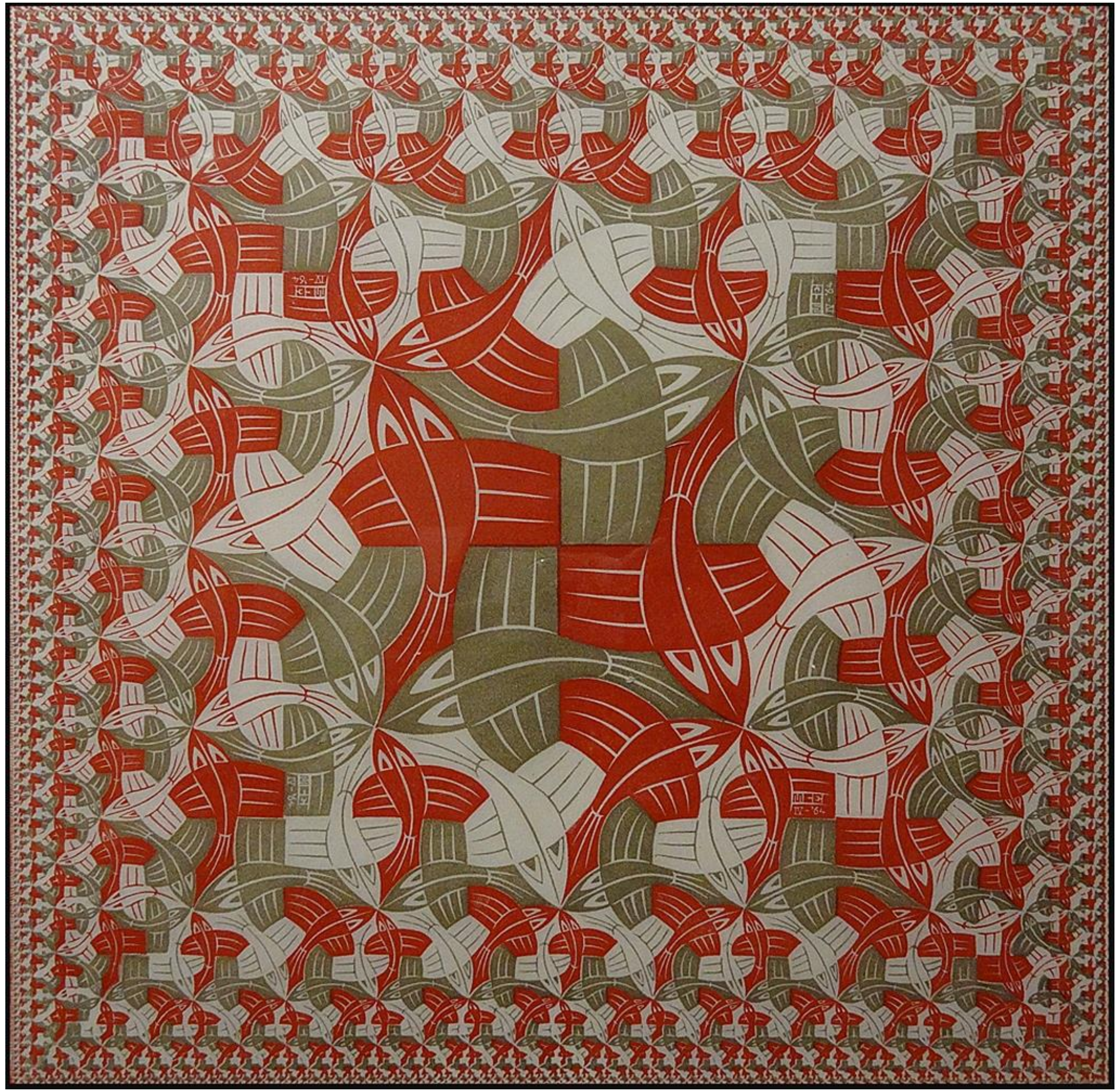
Schwarz-Christoffel transformation





Schwarz-Christoffel transformation

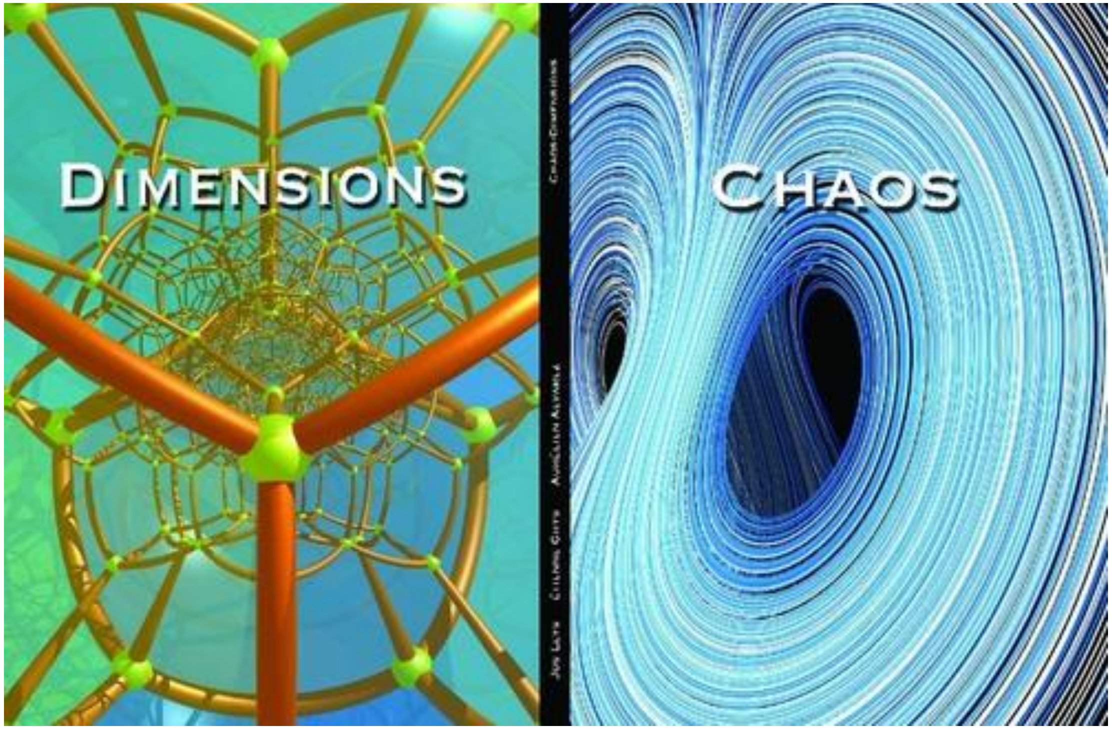
M.C.Escher
Vierkantlimiet,1964



“*Dimensions*” and “Chaos”

Ghys-Leys-Alvarez

- 2008 and 2013 film projects : both 2 hours of animation.
- Non-profit : Free download or watch online.
- Multiple commentary and subtitle languages.
  www.dimensions-math.org   www.chaos-math.org



Henri Paul de St Gervais

Analysis Situs

Topologie algébrique des variétés

Rechercher : >>

Entre 1895 et 1904, Henri Poincaré a fondé la topologie algébrique (alors appelée *Analysis Situs*) en publiant une série de six mémoires révolutionnaires. Ces textes fondateurs sont écrits dans le style inimitable de Poincaré : les idées abondent et... côtoient les erreurs. L'ensemble représente un peu plus de 300 pages de mathématiques exceptionnelles. Quelques historiens ou mathématiciens modernes ont cherché à analyser ces textes fondamentaux mais les articles sur ce sujet sont relativement courts, ne proposent pas une étude détaillée, et surtout sont destinés aux experts.

Pourtant, 130 ans plus tard, le contenu de ces mémoires reste non seulement d'actualité mais constitue un passage obligatoire pour tout apprenti topologue. Ce site a pour but de proposer un « objet pédagogique » d'une nature nouvelle permettant au lecteur d'acquérir une vision contemporaine du sujet à travers une approche historique.

Pour démarrer, il suffit de choisir l'une des trois « portes d'entrée » ci-dessous.

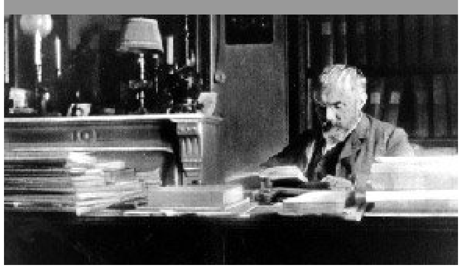
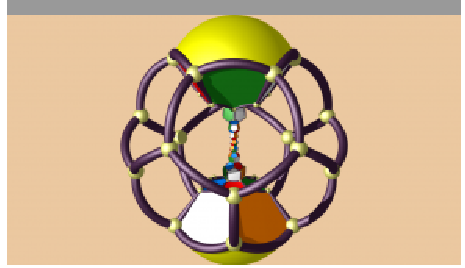
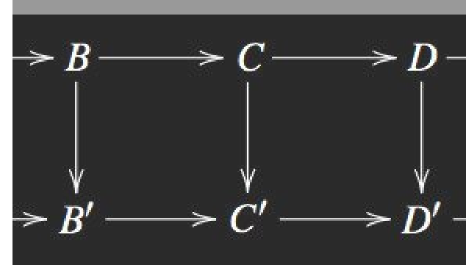
La première, *par les œuvres*, propose de commencer l'exploration de la topologie algébrique par les textes originaux de Poincaré ou des commentaires historiques.

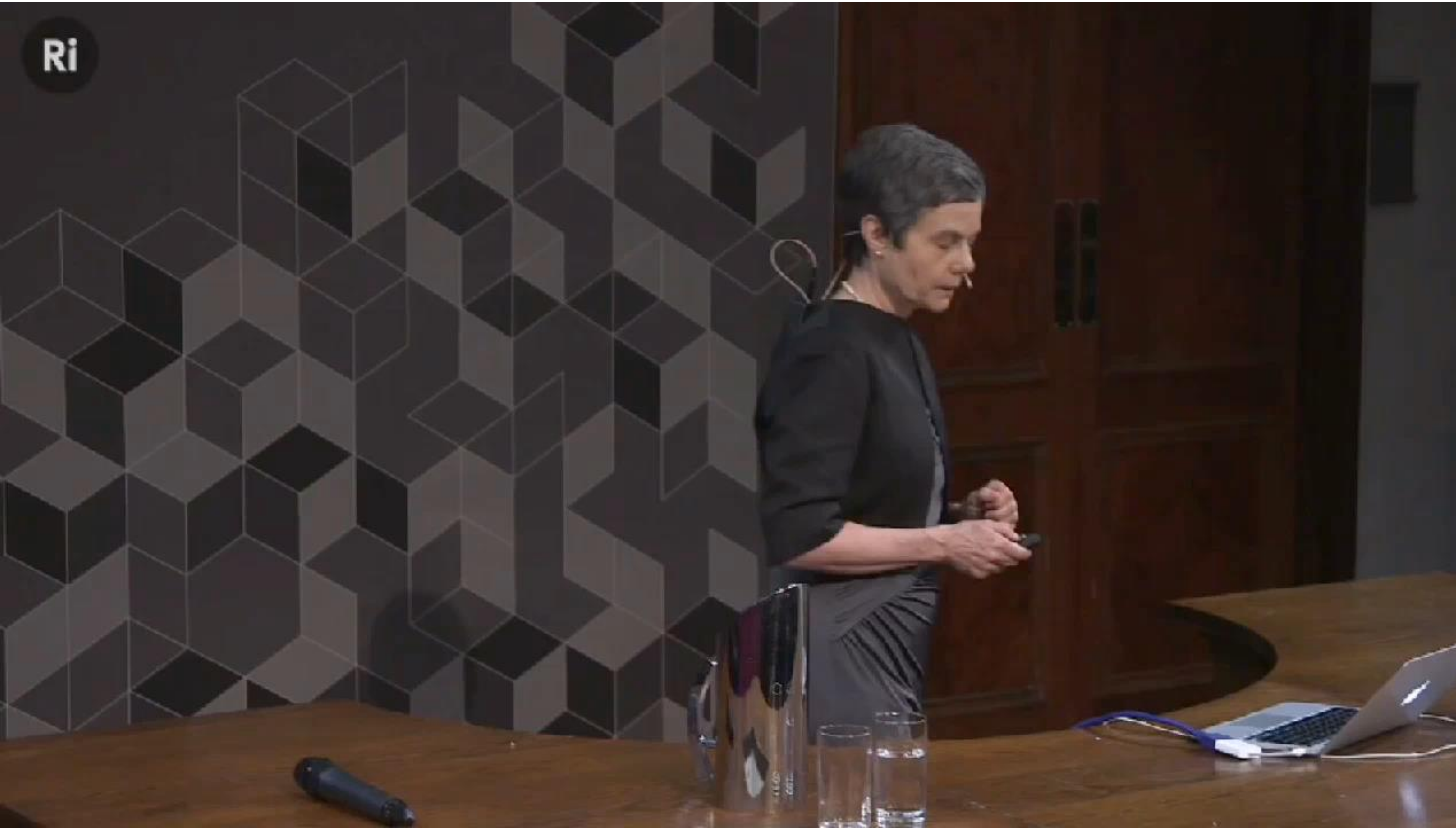
La seconde, *par les exemples*, propose de plutôt commencer par un choix d'exemples pour la plupart tirés des textes de Poincaré.

Enfin la troisième et dernière « porte d'entrée » propose un véritable *cours moderne* de topologie, niveau master, regroupé en grands thèmes selon le même « plan » --- ou la même anarchie --- que le texte source, mais dans lequel la présentation, le style, les démonstrations et les méthodes employées sont celles du 21ème siècle.

Ces trois parcours sont évidemment intimement liés, *laissez vous dériver au fil des nombreux liens...*. Enfin ces parcours sont émaillés de nombreuses animations et cours filmés que vous pouvez également retrouver sur notre chaîne youtube.

Henri Paul de St Gervais

Par les œuvres	Par les exemples	Cours moderne
		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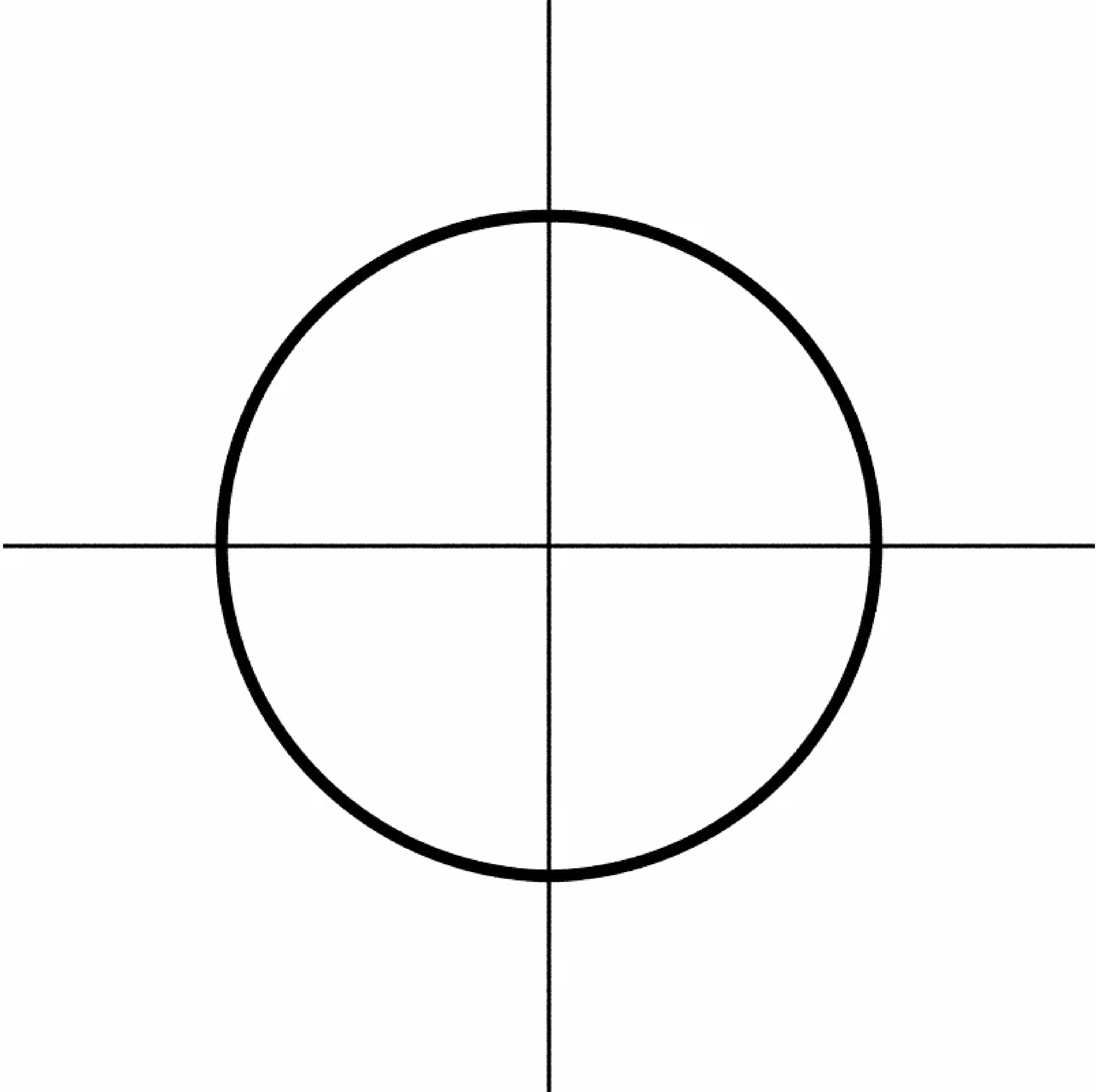

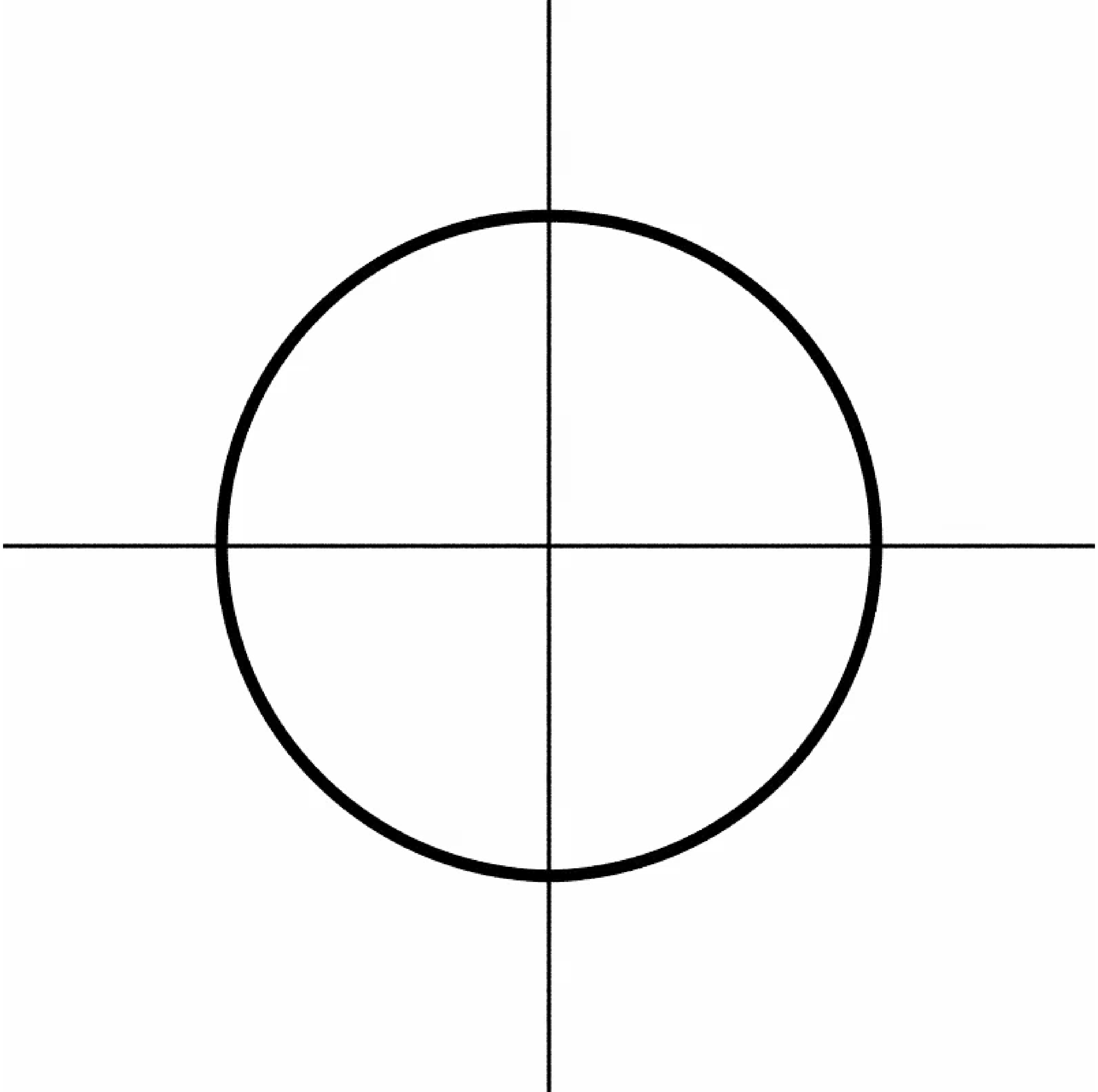

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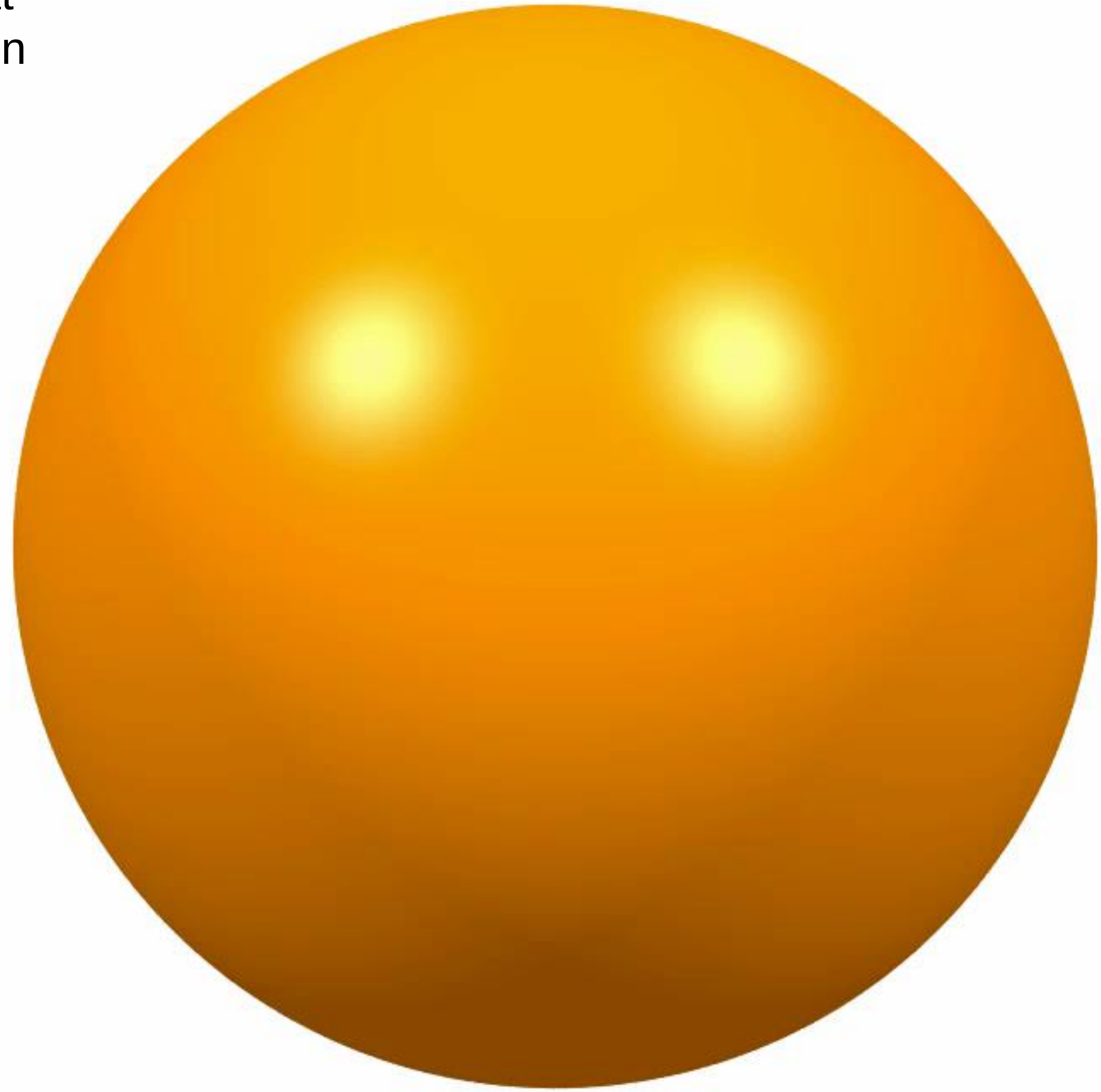

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