List of Abstracts BSSM 2024

Lundi-Monday

Guy Mélard

Title: Matrix polynomials and statistical analysis of time-dependent models for vector time series

Abstract: Matrix polynomials are useful in signal processing and the statistical analysis of time series models, in particular, to ensure stationarity and invertibility of vector autoregressive-moving average (VARMA) models. This is done using the companion matrix of a matrix polynomial and the examination of its eigenvalues.

An interesting approach consists of transforming the VARMA model into a VAR(1) model in a higher dimension. After a sketch of a real time series analysis, we define the models, the AR and MA polynomials, and introduce the statistical estimation problem. We do the same for models with time-dependent coefficients, leading to tdVARMA.

We show that the transformation to a tdVAR(1) model and the roots of their tdAR and tdMA polynomials make it easier to obtain simpler conditions than in the literature for the asymptotic properties of an estimator of the parameters. We conclude the presentation by showing other results for polynomials useful in the theory of time series.

Christoph Hertrich

Title: Using discrete mathematics to understand neural networks

Abstract: Artificial intelligence, and in particular neural networks, undoubtedly change our modern society. At the same time, we lack a thorough mathematical understanding of neural networks. In this talk I will introduce my favorite, surprisingly basic mathematical problem about neural networks, which remains unsolved until today. I will then give a glimpse on recent attempts to solve it using polytopes, which are beautiful geometric objects in discrete mathematics.

Sam Mattheus

Title: Random and algebraic constructions in extremal combinatorics

Abstract: We will talk about some problems in extremal combinatorics, and more precisely, extremal graph theory. In this branch of mathematics, one aims to maximize (or minimize) a certain graph parameter under some restrictions. An easy example would be: how many edges can a graph on n vertices without a cycle have? We will look at some specific questions, which despite many years of research, still do not have a conclusive answer (unlike the example question). On one side of these problems, the goal is to construct graphs with certain properties. We will give an overview of different flavors of construction methods, some older and some more recent, each with their own strengths and weaknesses.

Mardi-Tuesday

Leandro Vendramin

Titre: Convex sets and sums of squares

Abstract: Minkowksi's theorem in convex geometry is evidently pretty powerful, as it yields quick and easy proofs of Fermat's two square and Lagrange's four square theorems. It can also be used to prove the considerably harder Legendre's theorem on three squares.

Lenny Neyt

Titre: Nearly optimal time-frequency decay and how to detect it

Abstract: The Fourier transform of a signal reveals the amplitude associated with each frequency, offering two complementary perspectives for describing a signal: one can either specify the signal's strength over time or detail the amplitude for each frequency. The decay of the signal at infinity, whether in the time domain or the frequency domain, provides different insights. Decay in the time domain indicates that the signal's information is primarily concentrated within a finite time interval, while decay in the frequency domain suggests the absence of high oscillations and thus the signal's regularity.

A classical result due to Hardy demonstrates that the Gaussian achieves the optimal decay in both time and frequency. This talk focuses on those signals whose time-frequency decay is nearly optimal, closely approaching that of the Gaussian. We will particularly explore how such behavior can be identified by examining the decay of the signal's Hermite coefficients.

Paolo Saracco

Titre: From groups to groupoids, a path to internal symmetries

Abstract: We are used to associate the notion of symmetry with that of group or group action. However, even if groups are indeed sufficient to characterize homogeneous structures, there are many objects that exhibit what we clearly recognize as "symmetry", but whose description eludes the classical representation theory of groups.

It turns out that the symmetry, and hence much of the structure, of such objects can be characterized algebraically by using groupoids and partial group actions.

The aim of this talk is to introduce these structures, mostly through examples, and to explain how we can use them to capture more symmetries.

Mercredi-Wednesday

Julie De Saedeleer

Title: Une histoire de la cryptographie, ou comment devenir un bon agent secret.

Abstract: D'abord nous traverserons ensemble les siècles en regardant de près différents codes qui ont marqué l'histoire de la cryptographie. De la scytale de l'antiquité à certains codes actuels, en passant par la célèbre machine Enigma, les codes sont devenus indispensables et sont partout autour de nous.

Seconde partie: Un jeu de piste nous emmènera à travers les codes qui ont marqué l'histoire, amenant chaque joueur à comprendre les techniques de cryptographie et devenir un bon agent secret.

Quiz mathématique

Title: Quiz mathématique, le retour !

Abstract: L'équipe de la BSSM vous propose un nouveau quiz mathématique.

Jeudi-Thursday

Maxime Boucher

Title: A la pêche aux statistiques

Abstract: Vous avez déjà été invité à une journée pêche un dimanche mais en fin de journée, vous n'avez eu aucunes prises ? Peut-être n'y avait-il aucun poisson dans le lac, ou peut-être que si. Comment le savoir ? Nous verrons

qu'avec les statistiques nous pouvons estimer ce nombre de poissons grâce a la méthode de Capture-Recapture.

Léo Schelstraete

Title: Comment calculer en deux dimensions?

Abstract: Avez-vous déjà remarqué que, dans les mathématiques classiques, on calcule toujours en une dimension ? Par exemple, on peut multiplier 2 par 5 à *gauche* $(5 \cdot 2)$ ou à *droite* $(2 \cdot 5)$. De même, étant données deux fonctions f et g, on peut soit précomposer f par g (donnant $f \circ g$), soit postcomposer f par g (donnant $g \circ f$). Mais personne n'a jamais mis f *au-dessus* de g... Si ?

Cet exposé explore l'idée de calcul en deux dimensions à travers divers exemples, allant des circuits électriques à la théorie des nœuds, en passant par la physique quantique.

William Hautekiet

Title: Des carreaux et du papier peint

Abstract: Aujourd'hui on ne va pas droit dans le mur, on va d'abord le regarder! On va découvrir les différentes symétries que le papier peint peut posséder. Vous pouvez vous attendre à de beaux motifs et quelques extensions de groupes.

Vendredi-Friday

Gwenaël Joret

Title: How to decompose planar graphs?

Abstract: Planar graphs, graphs that can be drawn in the plane without edge crossings, are among the most natural and ubiquitous kinds of graphs. They appear in numerous real-life applications (think of road networks for instance). They also play a central role in an area of graph theory called "structural graph theory". For these reasons, it is interesting to study the structure of planar graphs: How can we decompose them into basic building blocks that are "simple"?

This is the topic of this talk. I will start with classic tools from the 1970s and 1980s such as the Lipton-Tarjan separators and Baker's decomposition technique, and then move on to some very recent results in the area obtained in the last 2-3 years. While planar graphs are as old as graph theory—they were already studied in the 1850s due to the 4 Color Conjecture—they are not yet fully understood, researchers are still discovering hidden structures and new theorems about these graphs today. This is an exciting research area to work in at the moment. In my talk I will give a gentle introduction to the area.

Justin Vast

Title: Matrices de Hadamard et représentations linéaires de groupes finis

Abstract: Dans cet exposé, nous dirons qu'une matrice carrée A possède la propriété (P) si chacun de ses coefficients est nul ou une racine de l'unité, et si $AA^* = A^*A = w\mathbb{I}$ pour un certain entier $w > 0$ appelé le poids de A.

Je décrirai comment construire de telles matrices à l'aide de la théorie des représentations des groupes finis. L'idée essentielle de cette construction consiste à se fixer une représentation ϱ d'un groupe fini G, et à fournir deux réalisations matricielles ρ_1 et ρ_2 de ϱ. On détermine ensuite une matrice de changement de base A satisfaisant $A\rho_1(g)A^{-1} = \rho_2(g)$ pour tout $g \in G$. Sous certaines conditions, la matrice A possède la propriété (P).

Comme corollaire, via la théorie des représentations de $\mathrm{GL}_2(\mathbb{F}_q)$, nous redécouvrirons à l'aide de cette méthode la célèbre construction de Paley pour les matrices de Hadamard.

Damien Galant

Title: The nonlinear Schrödinger equation on metric graphs

Abstract: Metric graphs are unidimensional mathematical structures of great interest both from the modelling and the mathematical point of view.

In this talk, I will focus on the study of the nonlinear Schrödinger equation on metric graphs. In particular, I will present the notion of "ground state" from calculus of variations and show how it allows to find solutions of the equation on metric graphs.