

The Kadison-Singer problem

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A problem in functional analysis dating back to 1959, originating in an obscure passage of Dirac's book on quantum mechanics, and solved in 2013 by 3 computer scientists... That makes a nice story to tell!

In 1959, R.V. Kadison (1925-2018) and I.M. Singer (1924) asked whether every pure state on the algebra of bounded diagonal operators on ℓ^2 , admits a unique extension as a pure state of $B(\ell^2)$. The positive answer was given in 2013 by A. Marcus, D. Spielman and N. Srivastava, after a series of translations of the original question, due to C. Akemann, J. Anderson, N. Weaver... The problem eventually boils down to estimating the largest zero of the expectation of the characteristic polynomial of a sum of independent random variables taking values in the set of rank 1 positive semi-definite matrices, in the algebra of complex n -by- n matrices.